SOIL BEHAVIOUR OPTIMISATION PROCEDURE OF COHESIONLESS SOIL AROUND PRESSUREMETERE

PRESFEREMETRE ÇATLAK TOPRAKLİK TOPRAKLĪĞİNİN TOPRAK DAVRANIŞI OPTİMİZASYON PROSEDÜRÜ

Younes ABED*1  Djillali AMAR BOUZID2  Ilhem TOUMI TOUMI3

ABSTRACT

This paper presents a methodology for identifying soil parameters that takes into account different constitutive equations. The procedure, applied here to identify the parameters of generalized Prager model associated to the Drucker & Prager failure criterion from a pressuremeter expansion curve, is based on an inverse analysis approach, which consists of minimizing the function representing the difference between the experimental curve and the curve obtained by integrating the model along the loading path in the in-situ testing. The numerical process implemented here is based on a finite element procedure. Some parameters effects on the simulated curve, examples identification and stresses path around pressuremeter probe are also presented.

Keywords: Pressuremeter test; finite element method; inverse analysis.

ÖZET


Anahtar Kelimeler: Basınçölçer testi; Sonlu elemanlar yöntemi; Ters analiz.

1. INTRODUCTION

During the three past decades, a tremendous progress has been made in the development of constitutive models, which are able to accurately reproduce the soil behaviour. However, the degree of complexity of these constitutive models (in many cases) inhibits their incorporation into general purpose numerical codes, thus restricting their usefulness in engineering practice. The successful application of in-situ testing of soils heavily depends on development of interpretation methods of tests. The pressuremeter test simulates the expansion of a cylindrical cavity and because it has well defined boundary conditions, it is more unable to rigorous theoretical analysis (i.e. cavity expansion theory) then most other in-situ tests.

*1 Younes ABED, Department of civil engineering, faculty of technology, university of Blida, Algeria, y.abed1967@univ-blida.dz
2 Djillali AMAR BOUZID, Department of civil engineering, faculty of technology, university of Blida, Algeria
3 Ilhem TOUMI, Department of civil engineering, faculty of technology, university of Blida, Algeria
Soil parameters can be identified from pressuremeter tests by using an inverse analysis of the cavity pressure–cavity strain relationship measured in the field. To perform this, the pressuremeter test is generally simulated as the expansion of an infinitely long cylindrical cavity inside an infinite uniform medium by means of closed-form analytical solutions (Moussavi et al., 2011) or approximate numerical models, such as the finite elements (Zanier, 1985; Carter et al., 1986; Cambou et al., 1990; Yu and Houlsby, 1991 and 1995; Fahey and Carter, 1993; Oliviari and Bahar, 1995; Cudmani and Oskinov, 2001; Hsieh et al., 2002; Levasseur, 2008; Shahin et al., 2008; Javadi and Rezania, 2009; Zhang et al., 2009; Levasseur et al., 2010; Liang and Sharo, 2010; Zhang et al., 2013; Abed et al., 2014 and Al-Zubaidi, 2015).

Engineers prefer closed-form analytical solutions than numerical models because the recourse to numerical methods is often time consuming and computationally non-effective. However, the use of approximate numerical models becomes the only possible alternative, in the case where it is difficult to obtain a closed-form solution due to the complexity of the soil constitutive model. Furthermore, in most elasto-plastic models, a relatively large number of parameter needs to be defined, which makes empirical curve fitting impractical so that the use of optimization algorithms for matching simulations to experiments becomes a necessity.

The aim of the present work is to develop an effective and consistent procedure for the identification of the engineering properties of soils by interpreting results from pressuremeter tests. In particular, the paper presents a methodology for the identification of parameter values in the Generalized Prager Model (GPM) (the reader is referred to the papers by Abed and Bahar, 2010 and Abed et al., 2014) of the engineering properties of soils by inverse analysis of the experimental cavity pressure–cavity strain curve measured during pressuremeter tests in the field. Two optimization techniques are presented; one is based on the simplex method of Nelder and Mead (Nelder and Mead, 1965) and the other is based on the decomposition of the pressuremeter curve.

2. THE PROPOSED MODEL

The GP-DP (which stands for Generalized Prager associated with Drucker and Prager Models) is an elasto-plastic constitutive model that is able to predict the nonlinear stress–strain behaviour of soils. Based on the use of a kinematic hardening function, a bilinear stress-strain behaviour can be simply described by means of a single internal variable (Figure 1a). Considering the idea of kinematic hardening and multiple combined internal variables, a piecewise linear stress-strain response can be described (Figure 1b). The stress-strain curve is approximated by linear segments along which the tangent shear modulus is assumed to be constant (Bahar et al., 1999).

![Figure 1. Stress-strain relationships: (a) bilinear; (b) piecewise linear.](image-url)
The model, based on a set of several single Prager models is defined by the compliance of \(n\) elastic elements and their associated \((n)\) yield surfaces. Eventually, a single linear spring elastic element of compliance \((f_0)\) and a single yield surface of threshold stress \((S_{\infty})\) can be introduced and connected in series to the collection of elements to represent the initial elastic strain and failure respectively. Thus the model is defined by \((2n + 2)\) parameters that can be represented by a discrete spectrum of compliance.

The adopted Kinematic hardening model, assumes that during the plastic deformation the loading surface translates as a rigid body in the stress space keeping the size, shape and orientation of the initial yield surface. The expression for the yielding surface is:

\[
f(\sigma_{ij}, \varepsilon_{ij}^p) = F(\sigma_{ij} - x_{ij}) - k^2
given where \(k\) is a constant and \(x_{ij}\) are the coordinates of the center of loading surface which changes as the development of plastic strains continues. The simplest version for determining the parameter \(x_{ij}\) is to assume a linear dependence of \(dx_{ij}\):

\[
dx_{ij} = C d\varepsilon_{ij}^p
\]

Where \(C\) is the work hardening constant, characteristic of a given material. Eq. (2) may be taken as the definition of the linear work hardening.

The plastic strain is given by the normality rule (standard material):

\[
(d\varepsilon_{ij}^p)^k = \left(\frac{\partial f}{\partial \sigma_{ij}}\right) d\lambda_k
\]

\[
(d\varepsilon_{ij}^p)^k = 0 \quad if \quad d\sigma_{ij} n_{ij} \leq 0
\]

The consistency condition means that the yield surface keeps the same radius, this helps to determine the plastic proportionality factor \(d\lambda\) with:

\[
d\lambda_k = \frac{1}{C_k} \left(\frac{\sigma_{ij}}{\partial \sigma_{kl}} \frac{\partial f}{\partial \sigma_{kl}}\right) d\sigma_{ij} n_{ij} > 0
\]

Where the underscript \(k\) denotes the \(k\)th element of chain;

\(C_k\) is the modulus of the \(k\)th element of the chain;

\(n_{ij}^k\) is the normal external vector to the yield surface.

3. YIELD FUNCTION AND PLASTIC FLOW RULE

The failure criterion associated with the proposed model is that of Drucker and Prager (Drucker and Prager, 1952). This criterion is based on the assumption that the octahedral shear stress at failure depends linearly on the octahedral normal stress through material constants. It was established as generalisation of the Mohr-Coulomb criterion for soil:

\[
\sqrt{J_2} = \alpha I_1 + K
\]
Where, the constants $\alpha$ and $K$ are material constants. 

$J_2$ is the second invariant of the stress deviator tensor and $I_1$ is the first invariant of the stress tensor, and are defined as follows:

$$I_1 = \sigma'_1 + \sigma'_2 + \sigma'_3 \quad (7)$$

$$J_2 = \frac{1}{6}[(\sigma'_1 - \sigma'_2) + (\sigma'_2 - \sigma'_3) + (\sigma'_3 - \sigma'_1)] \quad (8)$$

$\sigma'_1$, $\sigma'_2$, $\sigma'_3$, are the principal effective stresses. 

The parameters $\alpha$ and $K$ can be calibrated to make the Drucker-Prager criterion fit different parts of the Mohr-Coulomb criterion (Chen and Mizuno, 1990). The Mohr-Coulomb criterion and two different Drucker-Prager fits are shown in Figure 2.

The two fits are:

1. Triaxial extension fit: The triaxial extension corners of the Mohr-Coulomb criterion coincide with the Drucker-Prager criterion.
2. Triaxial compression fit: The triaxial compression corners of the Mohr-Coulomb criterion coincide with the Drucker-Prager criterion.

4. FINITE ELEMENT SIMULATION OF PRESSUREMETER TEST

In this work, the computed cavity pressure–cavity strain curves were obtained from the finite element simulation of an infinitely long cylindrical cavity expanding inside a cohesionless soil.

In order to apply the finite element method, the 1D expansion of an infinitely long cylindrical cavity is simulated. The segment $(r_0 - r_\text{cp})$ is divided into $n$ segments of unequal lengths (Figure 3). To ensure a good accuracy of stresses and strains, the mesh is relatively fine in the vicinity of the borehole. With growing distance to the probe, the elements increase progressively in size. With reference to the middle point of the Pressuremeter membrane, the
dimension of the finite element mesh used is $20r_0$ in width. The soil around the probe is modelled using a linear mesh (Abed et al., 2014).

**Figure 3.** Finite element mesh around pressuremeter borehole.

### 4.1 Solution of the problem in elastic zone

In elasticity, the analytical solution of the differential equation governing the expansion of a cylindrical cavity is given according to the elastic parameters (Clark, 1995):

\begin{align}
    d\varepsilon_r &= \frac{r_0}{r^2} dU_0 \\
    d\varepsilon_\theta &= -\frac{r_0}{r^2} dU_0 \\
    d\varepsilon_z &= 0
\end{align}  \tag{9}

\begin{align}
    d\sigma_r &= \frac{E}{1+\nu} \frac{r_0}{r^2} dU_0 \\
    d\sigma_\theta &= -\frac{E}{1+\nu} \frac{r_0}{r^2} dU_0 \\
    d\sigma_z &= 0
\end{align}  \tag{10}

The deformation occurs at a constant stress without volume changes (figure 4).

**Figure 4.** Yield zone around the expanding cavity

### 4.2 Solution of the problem in elasto-plastic zone

The behaviour law can be written by:

\[ d\varepsilon_{ij} = A_{ijkl} d\sigma_{kl} \]  \tag{11}

Where, the matrix $A_{ijkl}$ is function of the stress state and the loading path. It describes the soil behaviour for a loading increment (figure 4). It may be written as:
\[ A_{ij} = -\frac{\nu}{E} + \sum_{k=1}^{n} \frac{J_{e_k}}{S_{e_k}} \left( \alpha + \frac{1}{4f_{jz}} \right) \left( S_i - X_i(k) \right) \left( S_j - X_j(k) \right) \text{ For } i \neq j \quad (12) \]

\[ A_{ij} = -\frac{1}{E} + \sum_{k=1}^{n} \frac{J_{e_k}}{S_{e_k}} \left( \alpha + \frac{1}{4f_{jz}} \right) \left( S_i - X_i(k) \right) \left( S_j - X_j(k) \right) \text{ For } i = j \quad (13) \]

5. NUMERICAL PROGRAM

The determination of the parameters of the constitutive model from the pressure-meter test consists in solving the following inverse problem: to find a set of parameters which minimize the difference between the experimental data (here the pressuremeter curve defined as the applied pressure versus the cavity wall deformation) and the simulated curve. This problem is classically defined by an objective function which evaluates, for a given set of parameters, the discrepancy between the model prediction and the experimental data.

To achieve this purpose, a computer program baptised "Press-Sim" is developed in order to identify automatically the parameters of the proposed behaviour law. In order to evaluate the objective function, we use an algorithm that allows the decomposition of the pressuremeter curve in three parts.

The proposed program can be used in two ways:

Direct use: knowing the model parameters, the program allows the determination of the simulated curve;

Indirect use: from the experimental curve, the program allows the identification of the model parameters.

6. PARAMETERS EFFECT ON SIMULATED CURVE

The procedure followed in order to show some parameters effect on the simulated curve was by considering the sensitivity of some parameters. For this, the use of the numerical program allows to get a simulated pressuremeter curve. To carry out this analysis, the experimental pressuremeter curve deduced from the thick cylinder tests realized on Hostun sand at ENPC (Ecole des Ponts ParisTech) by Dupla (Dupla, 1995) has been used.

6.1 Dilatancy effect on the pressuremeter curve

We present in figure 5 the effect of dilatancy on the value of the limit pressure. We remark that an increase of dilatancy factor \( \gamma \) leads to an increase in the limit pressure.
6.2 Effect of the consolidation pressure on the pressuremeter curve

Figure 6 presents the consolidation pressure effect on the pressuremeter curve. The corrected limit pressure is almost proportional to the pressure of consolidation, which is in accordance with experimental observation (Dupla, 1995).

6.3 Influence of the finite length of the pressuremeter probe

To study the influence of the finite length of the pressuremeter probe, a series of simulation of slenderness ratio (L/D = 5, 10, 15 and 20). It appears in figure 7 that the geometry of the
probe has a considerable influence on the pressuremeter curve. This difference is inversely proportional to the ratio L/D which is consistent with the experiment (Zanier, 1985)

![Graph showing the effect of slenderness ratio on pressuremeter curve](image)

**Figure 7.** Slenderness ratio effect

### 6.4 Stresses path around pressuremeter

The proposed method allows visualising stresses path in all the point of a chosen discretization. Figure 8 indicates the distribution of the main stresses at the end of loading for the self-boring pressuremeter test undertaken on the Fos sand in France. Three different areas of soil from the borehole wall to the infinite radius are considered. Plasticity appears between the radial stress $\sigma_r$ and circumferential stress $\sigma_\theta$ in the horizontal plane. This first plastic area extends between the radius a (borehole wall) and b (external radius of the plastic area). As shown by Wood & Wroth (Wood & Wroth, 1977) and Monnet (Monnet, 2007), plasticity may also appear in the vertical plane between the vertical stress $\sigma_z$ and circumferential stress $\sigma_\theta$ in an area between the radius b and radius c (external radius of both plastic areas). An elastic area extends beyond the radius b.
Figure 8. Distribution of main stresses at end of loading

7. SIMULATION EXAMPLES

7.1 Mini-pressuremeter tests carried out at IMG, Grenoble (France)

Simulations were performed on experimental curves obtained from the pressuremeter tests carried out in the calibration chamber on the Hostun sand at IMG (Institut Mécanique de Grenoble, France). We note that the gap between the experimental curve and the theoretical curve for the different simulations does not exceed 5% (figures 9a-9c). Tables 3 and 4 respectively show the comparison between model parameters and friction angle comparison with other methods and tests.

Figure 9a. H161P27 test (Boubanga, 1990)
Model parameters for H161P18 test:
- $E = 45$ MPa
- $A = 0,001$
- $\phi = 40^\circ$
- $P_0 = 18$ kPa
- $\gamma = 0,003$
- $\gamma_d = 16,1$ kN/m$^3$

**Figure 9b.** H161P18 test (Boubanga, 1990)

Model parameters for H157P27 test:
- $E = 15$ MPa
- $A = 0,01$
- $\phi = 35^\circ$
- $P_0 = 27$ kPa
- $\gamma = 0,001$
- $\gamma_d = 15,7$ kN/m$^3$

**Figure 9c.** H157P27 test (Boubanga, 1990)

Model parameters for H155P18 test:
- $E = 13$ MPa
- $A = 0,015$
- $\phi = 36^\circ$
- $P_0 = 18$ kPa
- $\gamma = 0$
- $\gamma_d = 15,5$ kN/m$^3$

**Figure 9d.** H155P18 test (Boubanga, 1990)
Figure 9c. H155P34 test (Boubanga, 1990)

**Figure 9.** Comparison between simulated and experimental curve for mini-pressuremeter test realized on Hostun sand in IMG, France

### Table 1. Model parameters

<table>
<thead>
<tr>
<th>Test</th>
<th>$\gamma_d$ (kN/m$^3$)</th>
<th>$P_0$ (kPa)</th>
<th>E (MPa)</th>
<th>A</th>
<th>$\phi'$ (°)</th>
<th>$\gamma$</th>
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<td>0.0008</td>
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</tr>
</tbody>
</table>

In Table 1, we show the model parameters deduced from the different simulations. Comparison of the friction angles in Table 2 shows that all methods including triaxial test provide friction angles included in a thin range.

### Table 2. Results comparison

<table>
<thead>
<tr>
<th>Test</th>
<th>Duncan</th>
<th>Lade</th>
<th>ECL</th>
<th>Proposed method</th>
<th>Triaxial test</th>
</tr>
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</tbody>
</table>
8. CONCLUSION

- The objective of this study was to develop an inverse method of identifying the behavior parameters of generalized Prager model, associated with the failure criterion of Drucker and Prager.
- The resolution of the inverse problem of the cylindrical cavity expansion in an elastoplastic medium showed the ability of the method to determine soil parameters wherever the generalized Prager model was used.
- Taking into account the dilatancy effect in this study has shown its effect on the pressuremeter curve, the limit pressure increases with the dilatancy of the material.
- The corrected limit pressure is almost proportional to the pressure of consolidation, which is consistent with experimental observation.
- This study showed that the assumption of plane strain is verified with the increase of slenderness of the probe.
- The distribution of main stresses at the end of loading shows that the plasticity may occur between the radial stress $\sigma_r$ and the circumferential stress $\sigma_\theta$ and between the vertical stress $\sigma_z$ and the circumferential stress $\sigma_\theta$.
- The difference between simulated curves and experimental curves for the different simulations is mostly tolerable.
- The values of the friction angle are quite close to those deduced from the triaxial tests and other methods.

9. REFERENCES

[14] Javadi A. A. and Rezania M., Applications of artificial intelligence and data mining techniques in soil modeling, Geomechanics and Engineering, 01(2009), 53-74