ÇELİK YAPILARDA DOĞRUSAL OLMAYAN MODELLEMENİN ESASLARI

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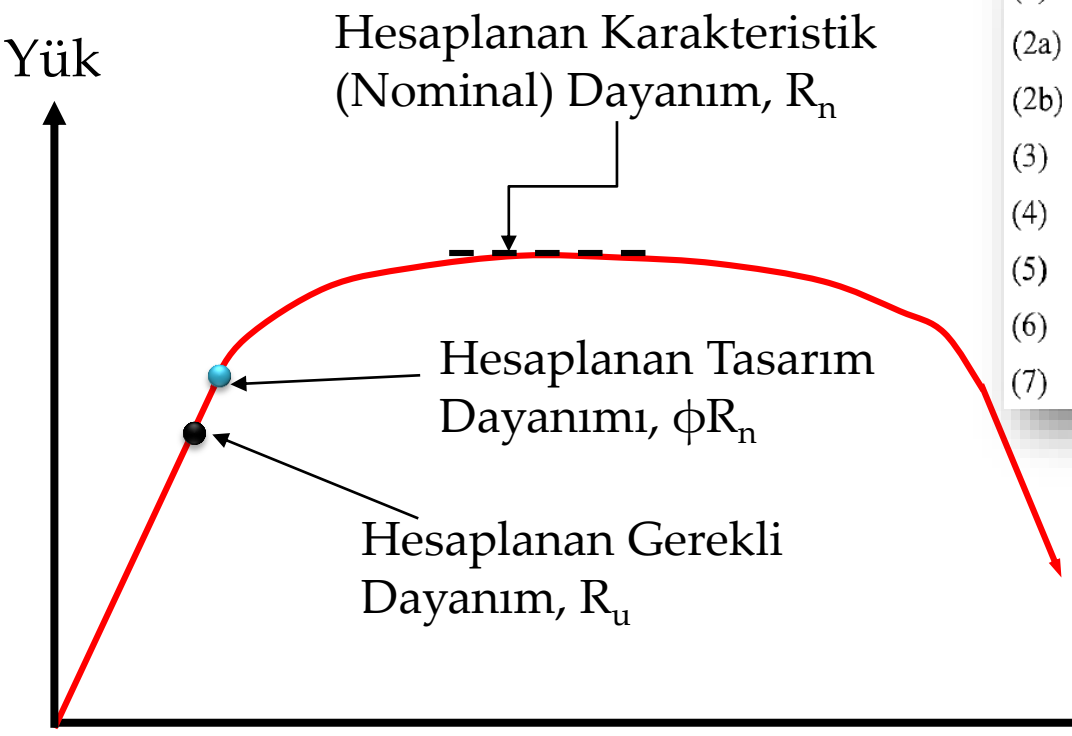
15 Kasım 2026, İstanbul

İçerik

1. Tasarım Yaklaşımları
2. Doğrusal Olmayan Davranış Modelleri
3. Statik İtme Analizi Örnekleri

Dayanıma Göre Tasarım

Dayanıma Göre Tasarım



(1) $1.4G$

(2a) $1.2G + 1.6(Q_r \text{ veya } S \text{ veya } R)$

(2b) $1.2G + 1.6Q + 0.5(Q_r \text{ veya } S \text{ veya } R)$

(3) $1.2G + 1.6(Q_r \text{ veya } S \text{ veya } R) + (Q \text{ veya } 0.8W)$

(4) $1.2G + 1.0Q + 0.5(Q_r \text{ veya } S \text{ veya } R) + 1.6W$

(5) $1.2G + 1.0Q + 0.2S + 1.0E$

(6) $0.9G + 1.6W$

(7) $0.9G + 1.0E$

$$R_u \le \phi R_n$$

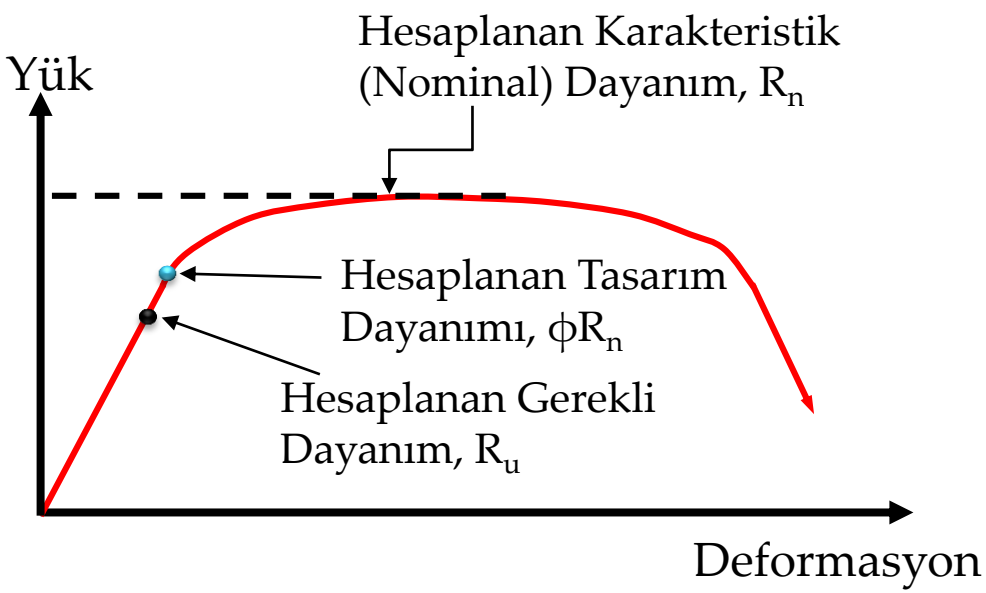
R_u : YDKT yük birleşimleri altında hesaplanan gerekli dayanım.

R_n : Karakteristik dayanım.

ϕ : Dayanım katsayısı.

ϕR_n : Tasarım dayanımı.

Dayanıma Göre Tasarım



Talep ≤ Kapasite

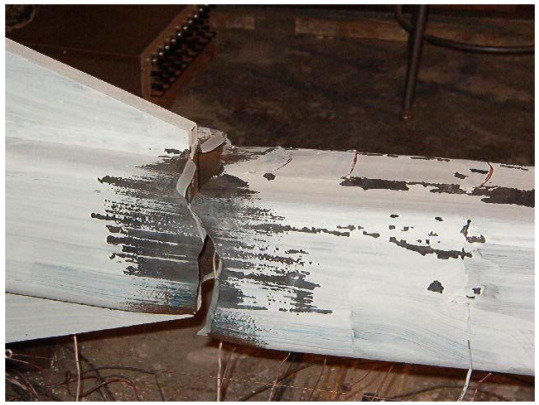
- Talepler doğrusal analiz yöntemi ile hesaplanır.
- Olası sınır durumları gözeterek eleman kapasiteleri belirlenir.
- Her bir eleman için talep/kapasite oranının kontrol edilir.
- Talebin kapasiteden yüksek olması halinde yeniden tasarım.

Dayanıma Göre Tasarım

Dayanım esaslı tasarımda göz önüne alınan *sınır durumlardan* bazıları:

1. Kayıpsız enkesitte akma sınır durumu;
2. Basınç elemanında global burkulma;
3. Etkin net enkesitte kırılma;
4. Plastik mafsal oluşumu;
5. Yanal burulmalı burkulma;
6. Yerel burkulma sınır durumları;
7. Kesmede akma, kırılma, blok kırılma sınır durumları;
8. Kaynakta kırılma;
9. Bulonda kesme kırılması;
10. Taban levhasında ezilme;
11. Ankrajlarda aderans kaybı;
12. Kesme çivilerinde ezilme;
13. Başlıkta yerel eğilme sınır durumu;
14. Gövdede yerel akma sınır durumu.



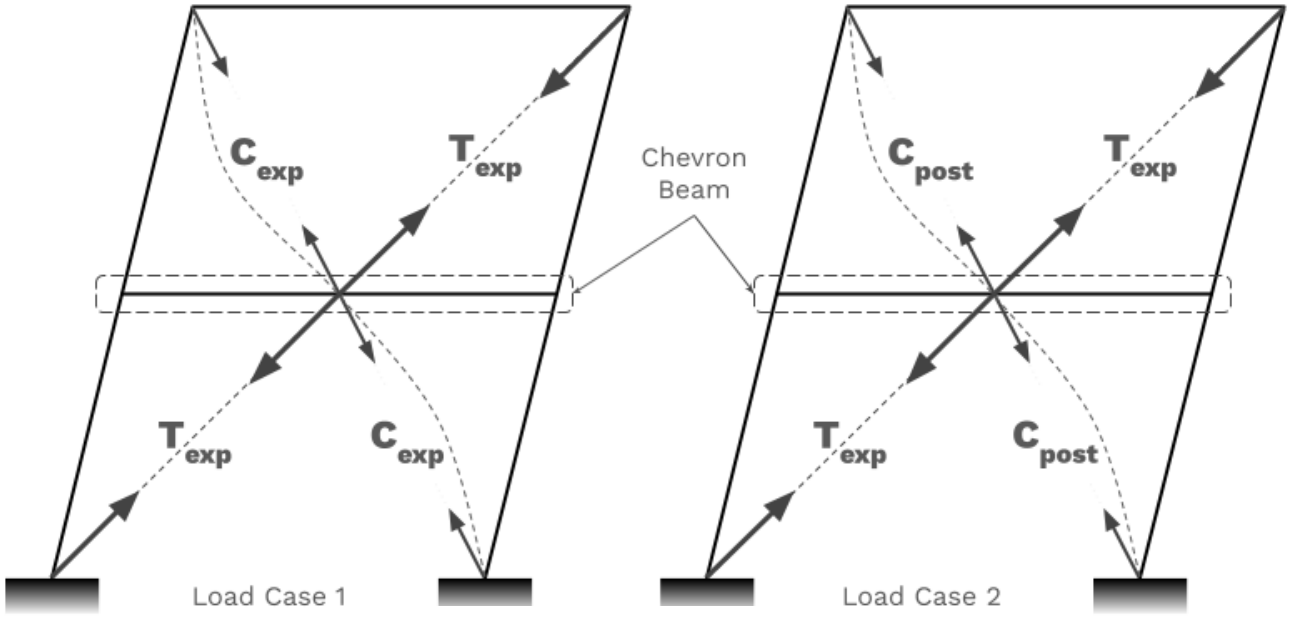


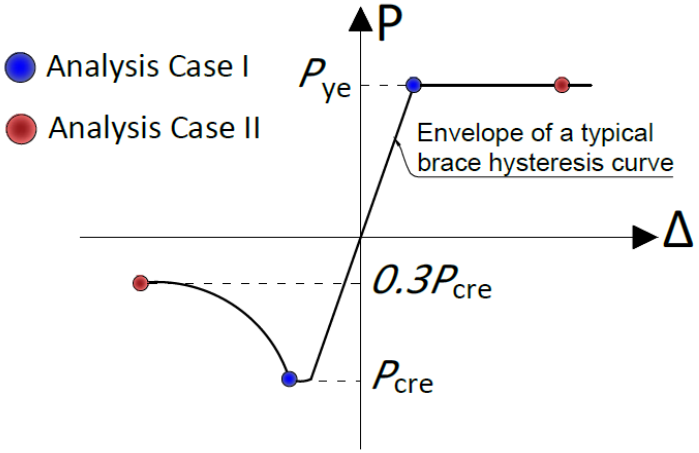




Dayanıma Göre Tasarım

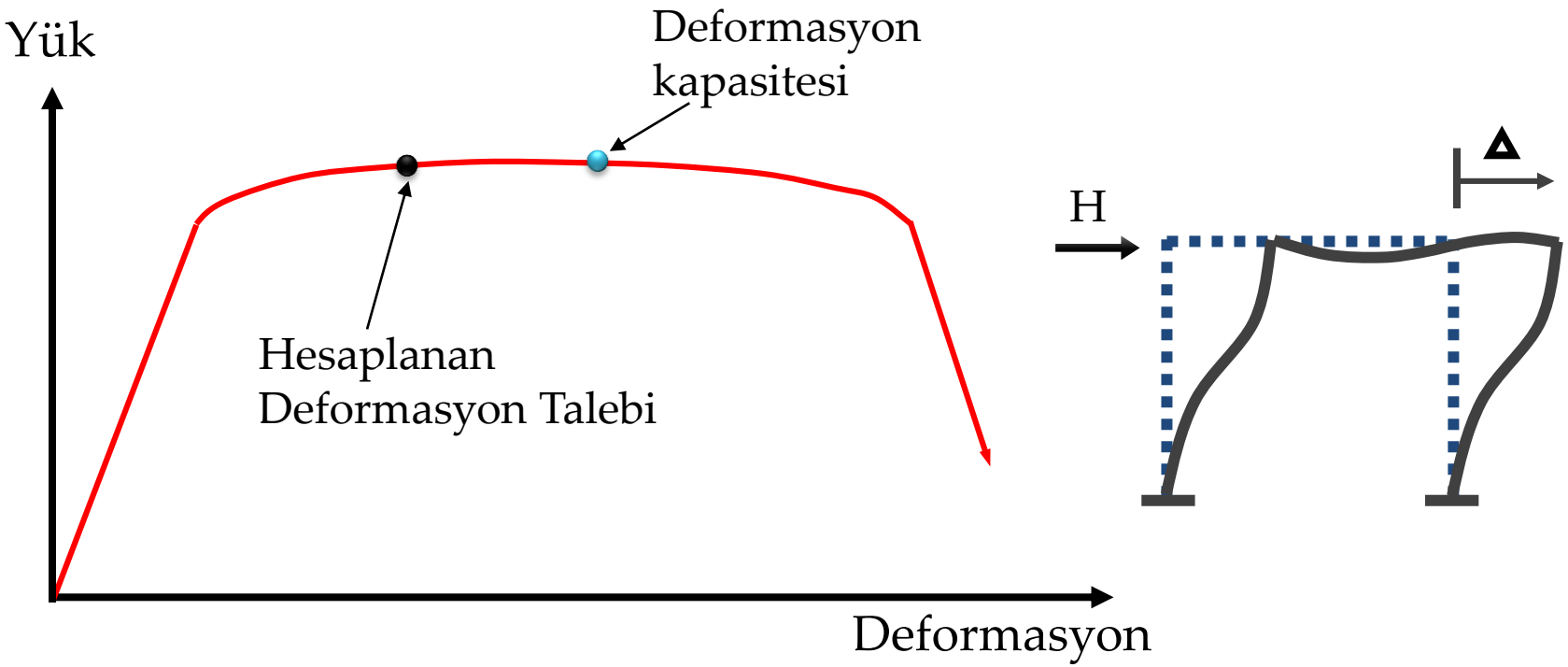
Dayanım esaslı tasarımda doğrusal olmayan davranış dolaylı olarak göz önüne alınır:





Şekildeğiştirmeye Göre Tasarım

Şekil Değiştirmeye Göre Tasarım

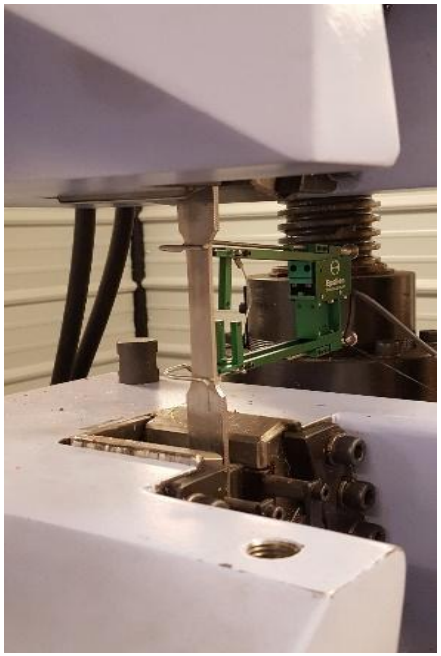


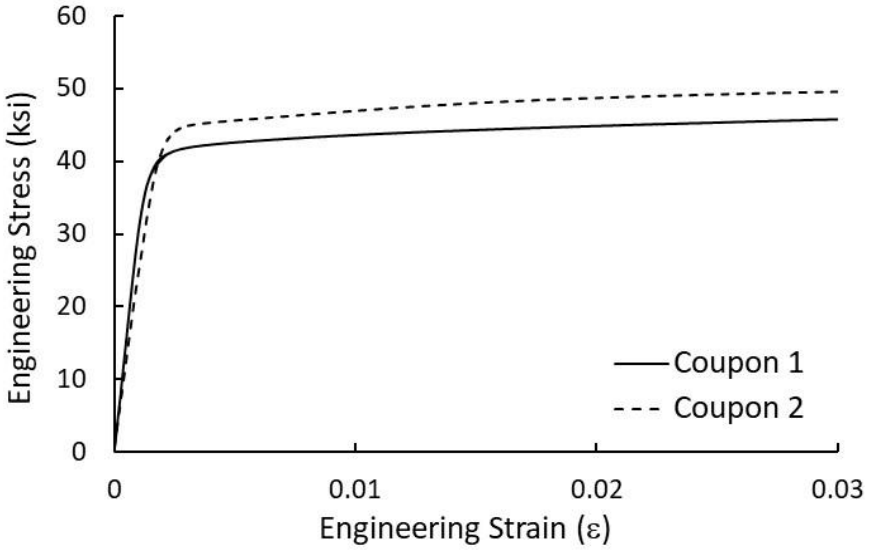
- Yer değiştirme/deformasyon talepleri doğrusal olmayan analiz yöntemleri ile tahmin edilir.
- Olası sınır durumları gözeterek kapasitenin belirlenmesi.
- Her bir eleman ve yapının tümü için talep/kapasite oranının kontrol edilmesi.

Şekil Değiştirmeye Göre Tasarım

Şekil değiştirmeye göre tasarım adımları:

1. Malzeme düzeyinde dayanım ve davranışın belirlenmesi;

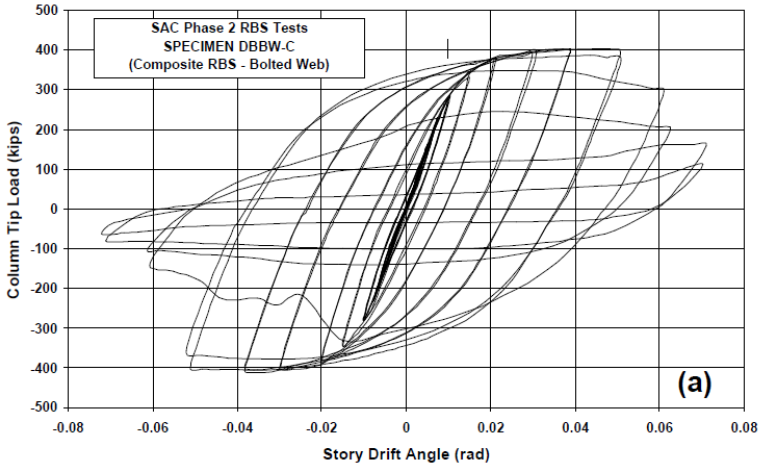


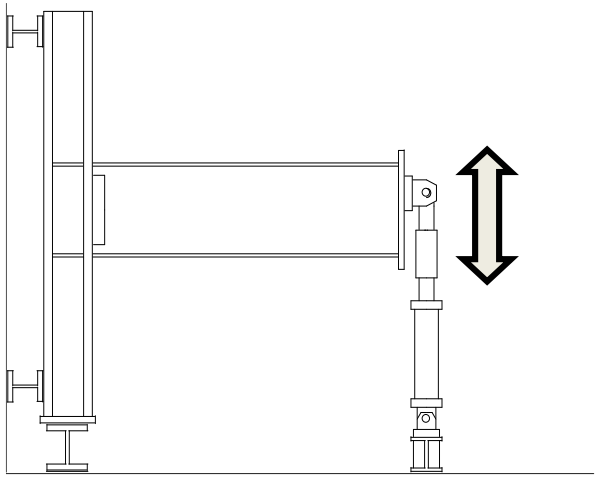


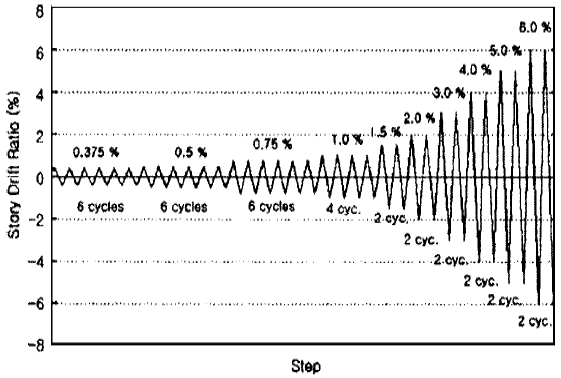
Şekil Değiştirmeye Göre Tasarım

Şekil değiştirmeye göre tasarım adımları:

2. Elastik ötesi davranış göstermesi beklenen sünek yapısal elemanların deformasyon kapasitelerinin tanımlanması (yük-deformasyon ilişkilerinin belirlenmesi);





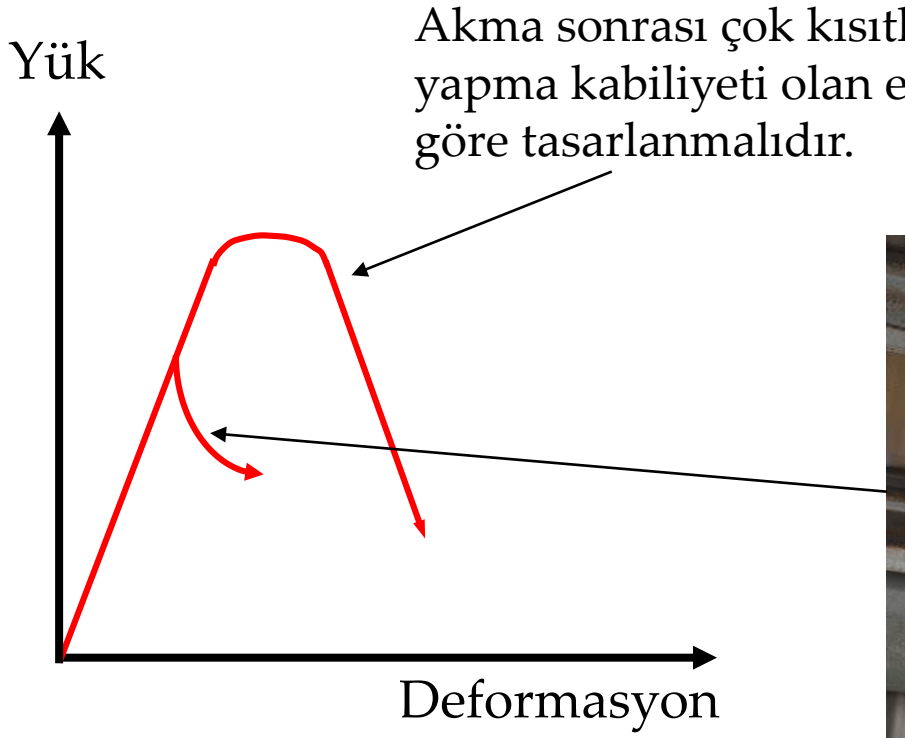




Şekil Değiştirmeye Göre Tasarım

Şekil değiştirmeye göre tasarım adımları:

3. Elastik kalması istenen veya gevrek davranışa sahip yapısal elemanların dayanım kapasitelerinin tanımlanması;

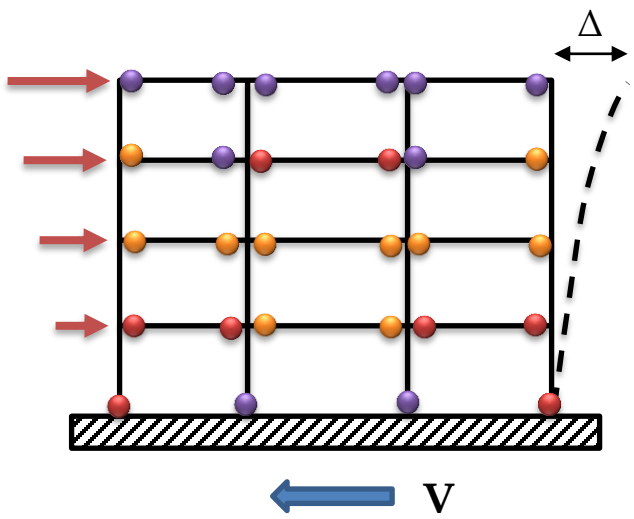


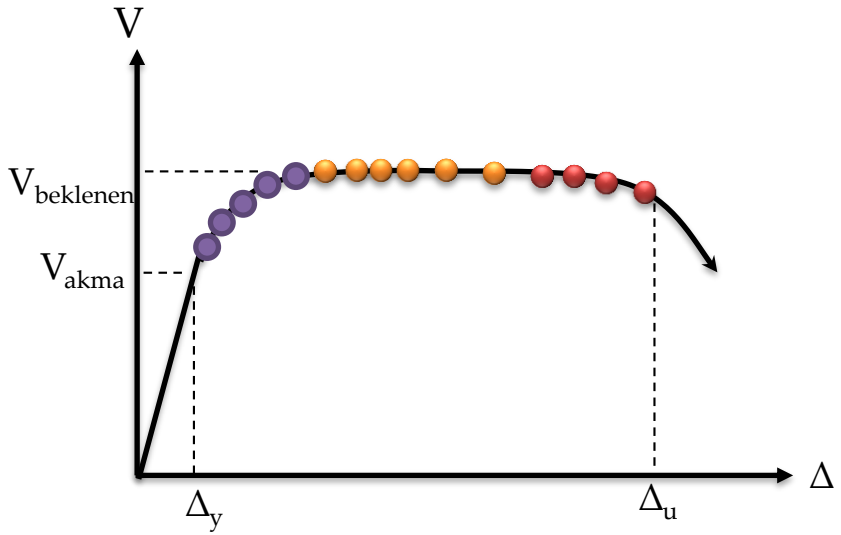


Şekil Değiştirmeye Göre Tasarım

Şekil değiştirmeye göre tasarım adımları:

4. Doğrusal olmayan analiz yöntemleriyle dayanım ve yer/şekildeğiştirme taleplerinin belirlenmesi.





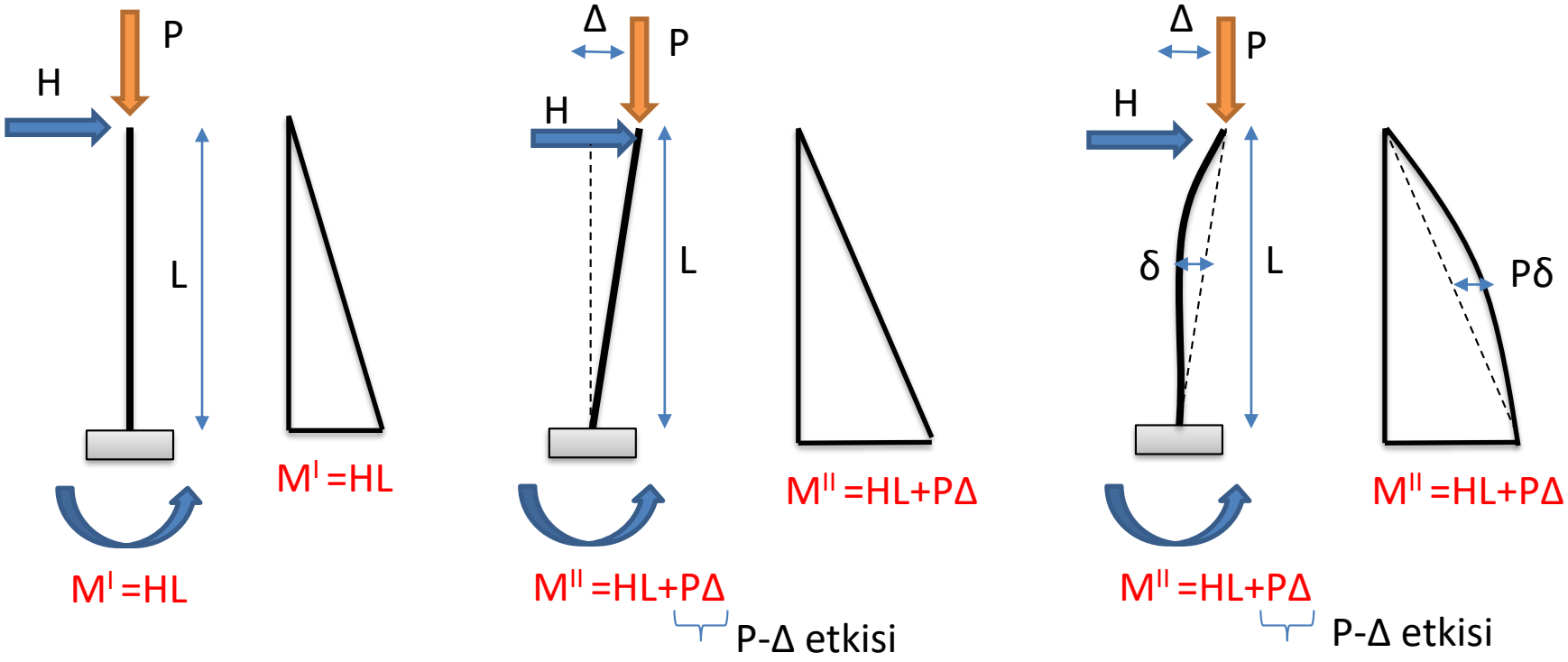
Talep ≤ Kapasite

Doğrusal Olmayan Davranış Modelleri

Doğrusal Olmayan Davranış

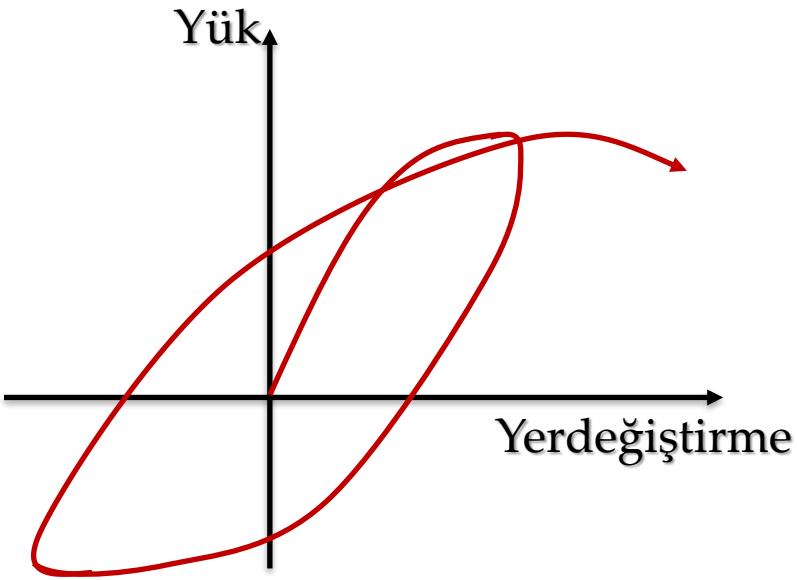
Doğrusal olmama hali:

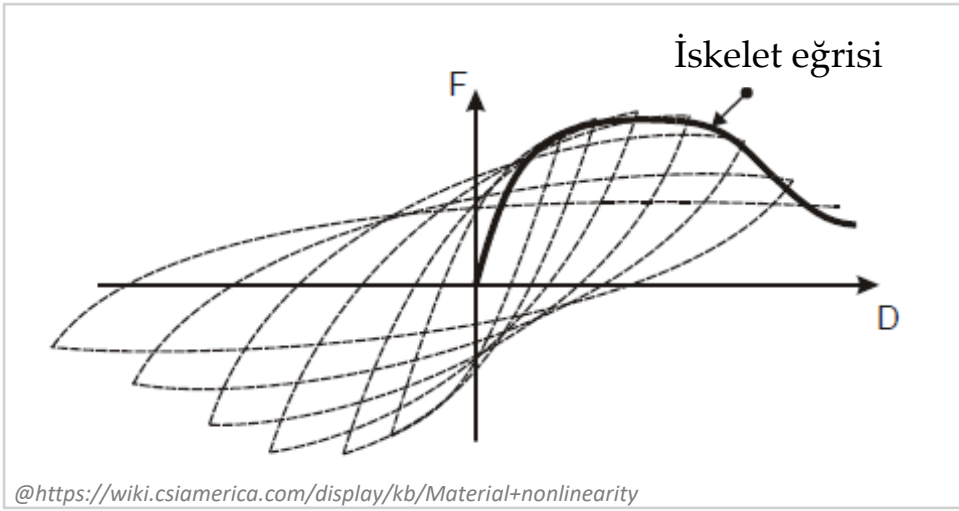
1. Malzemede elastik ötesi davranış;
2. Yapı ve yapısal elemanların yer ve/veya şekil değiştirmesinden kaynaklanan geometrik doğrusal olmama hali;
3. Doğrusal olmayan mesnet şartları.



Çevrimsel Davranış

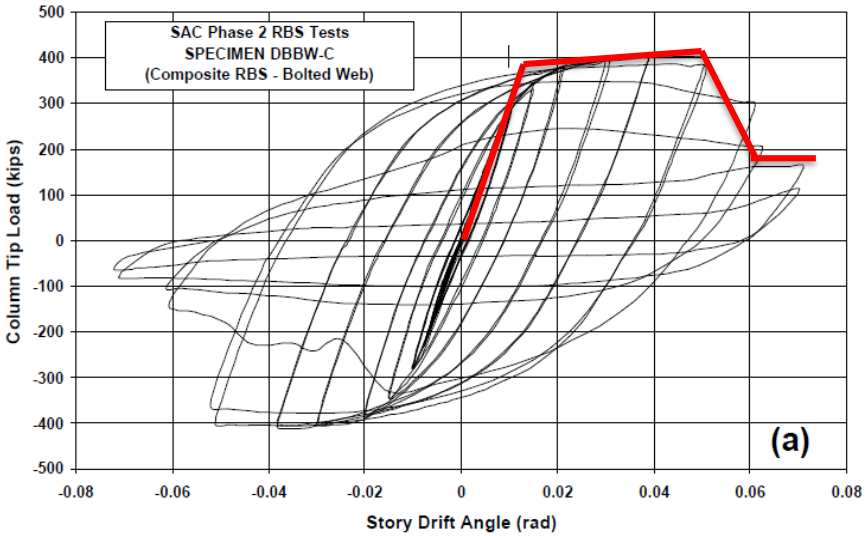
Doğrusal olmayan davranış, ancak doğrusal olmayan davranış göstermesi beklenen elemanların çevrimsel davranışının «olabildiğince» doğru modellenmesiyle tahmin edilebilir.



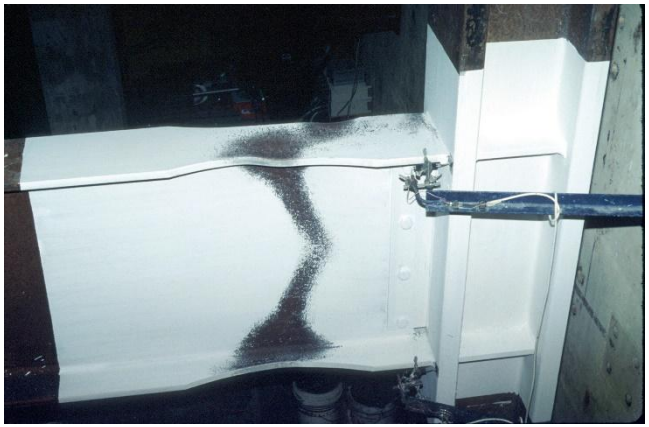


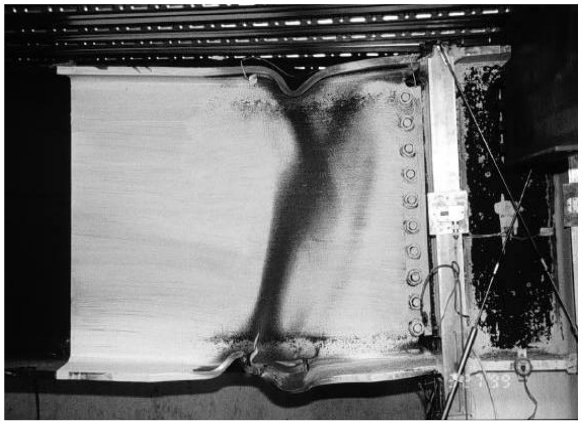
@https://wiki.csiamerica.com/display/kb/Material+nonlinearity

Kolon-Kiriş Birleşimi







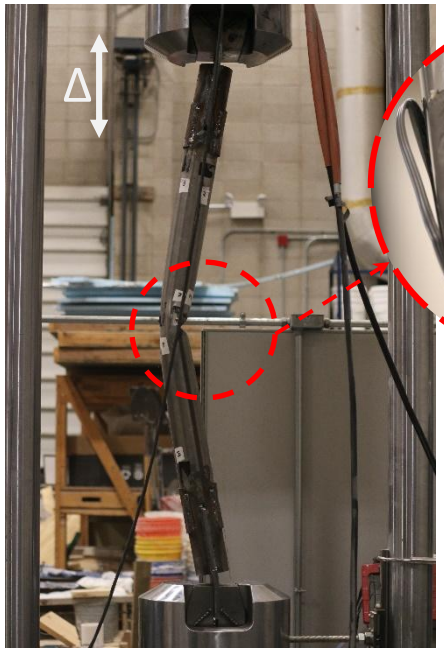


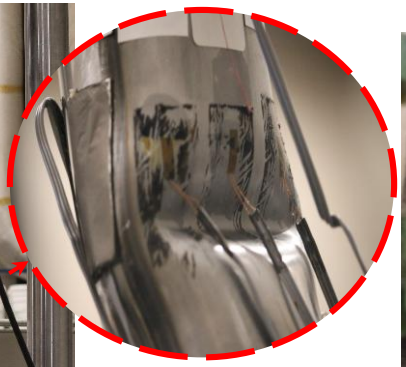


Akma → Yerel Burkulma → Başlıkta Kırılma

Konvansiyonel Çapraz

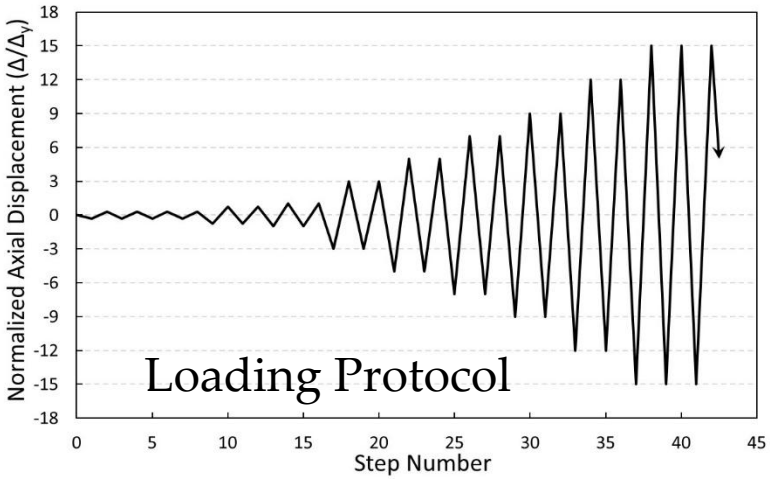


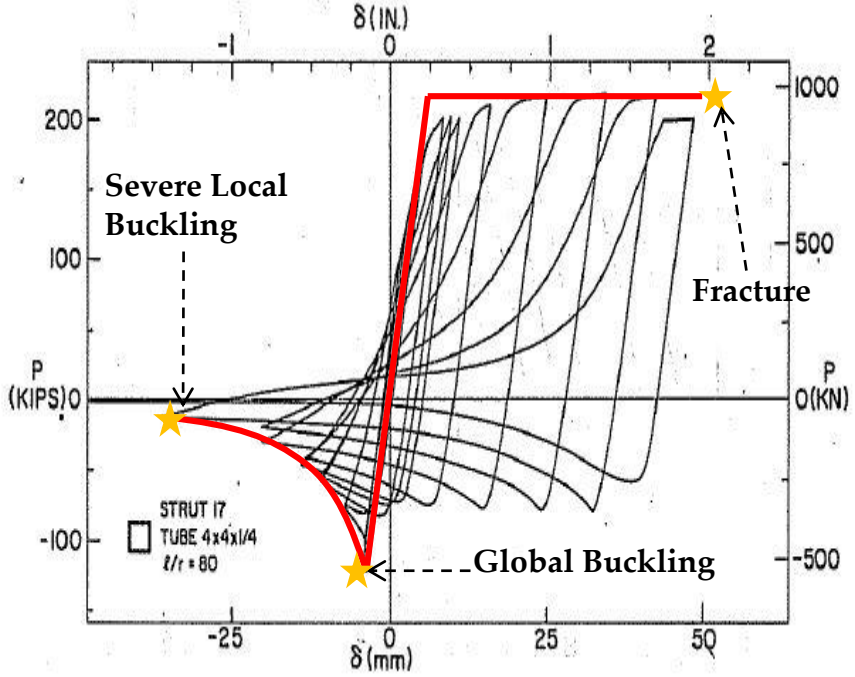






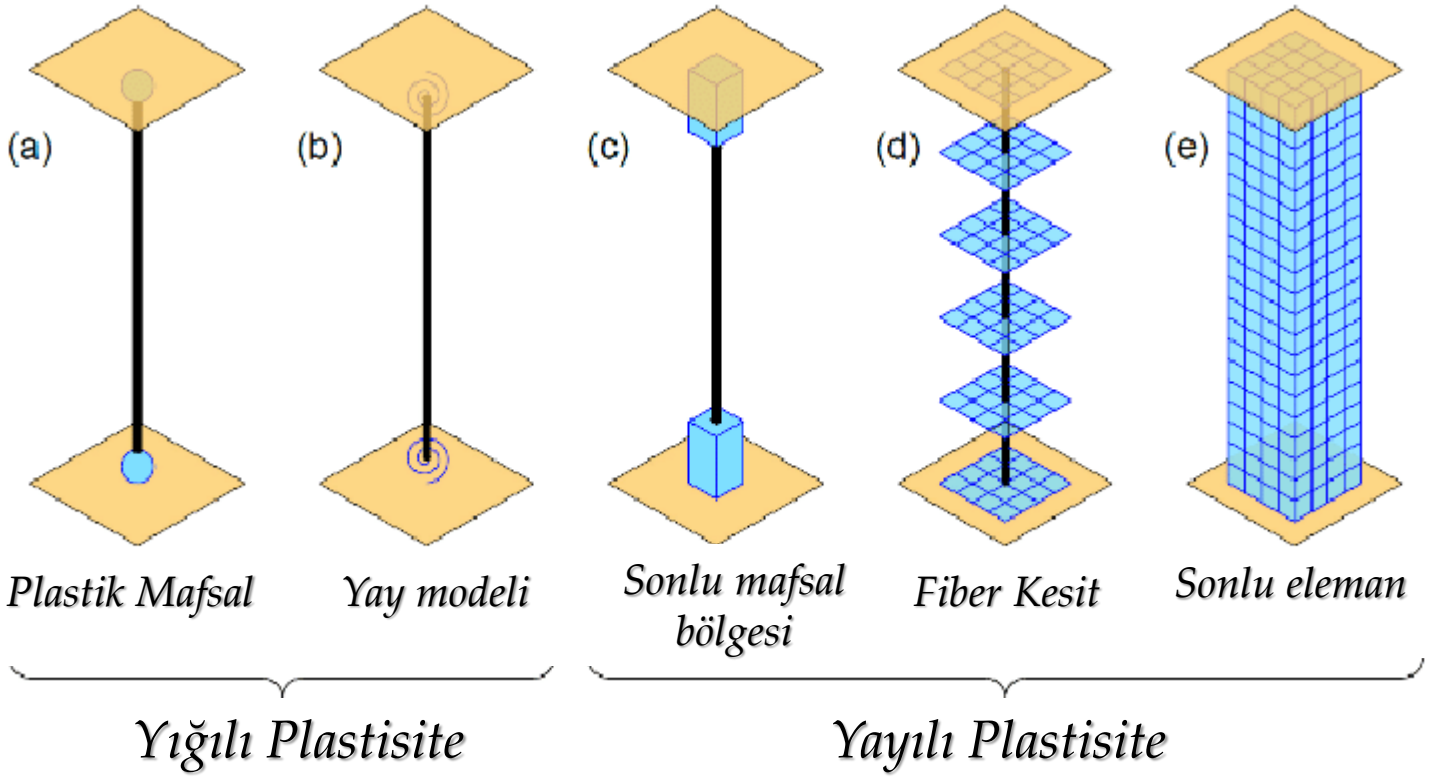
Brace test





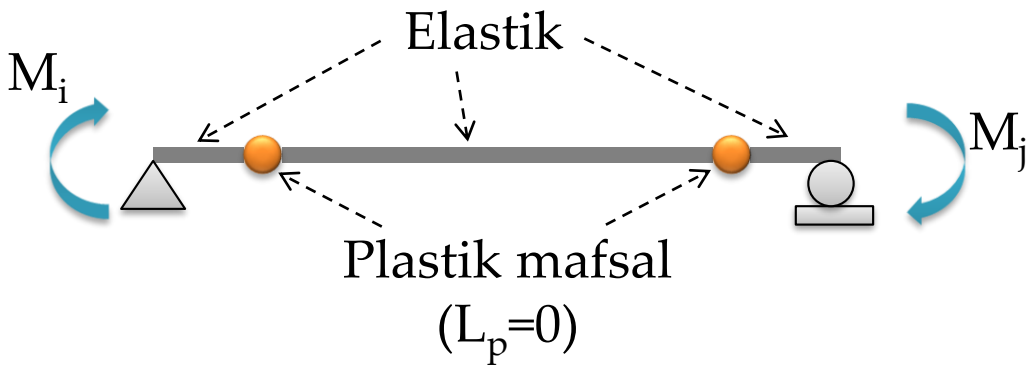
Doğrusal Olmayan Davranış Modelleri

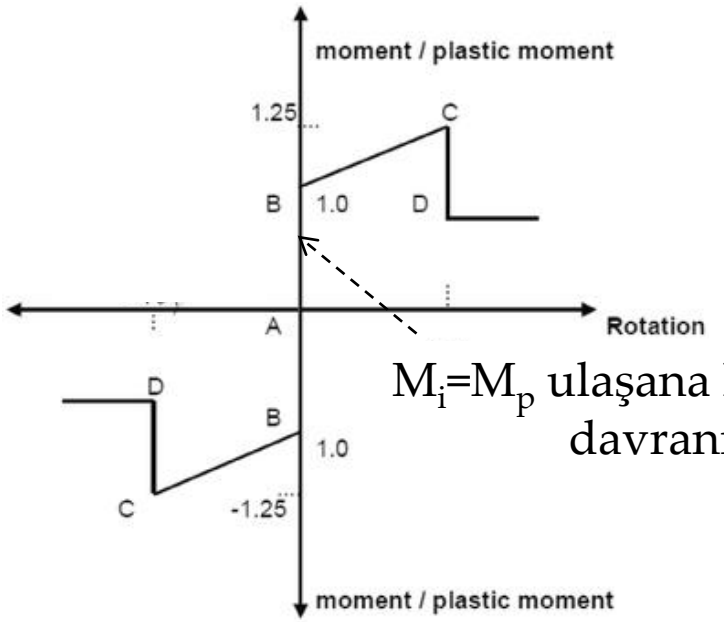
İdealleştirilmiş doğrusal olmayan davranış modelleri



Doğrusal Olmayan Davranış Modelleri

1. Yığılı Plastik Davranış Modeli (Plastik Mafsal)



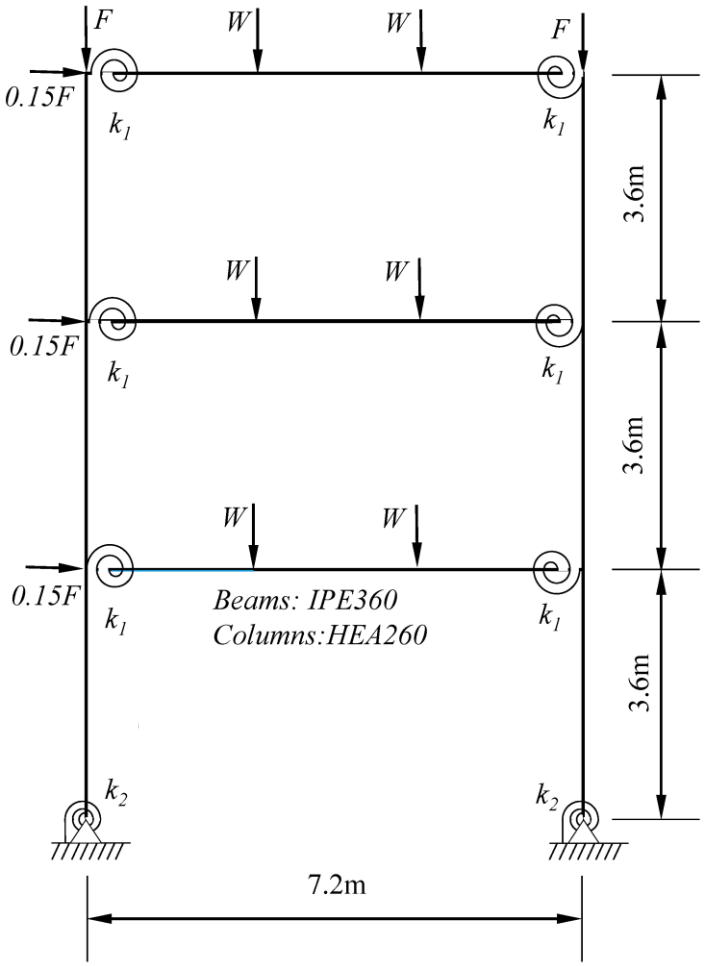


- Tüm doğrusal olmayan davranışın tek bir noktada (sıfır uzunluklu plastik mafsalda) toplandığı varsayılır.
- Deney sonuçlarına bağlı fenemonolojik model.

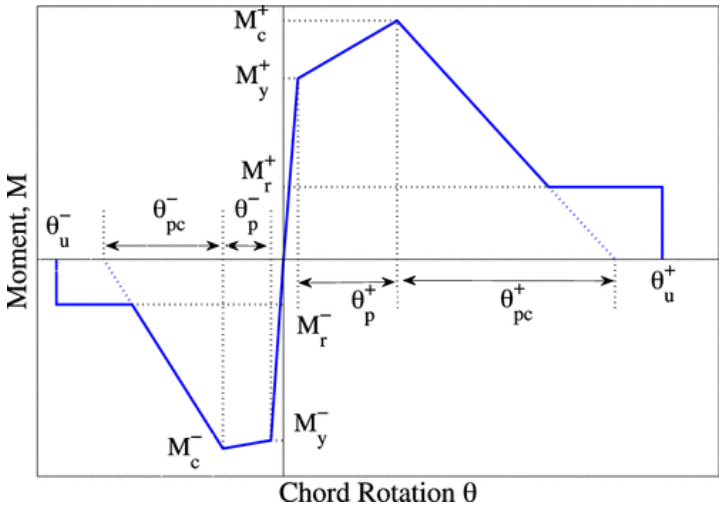
M_i=M_p ulaşana kadar rijit davranış

Doğrusal Olmayan Davranış Modelleri

2. Yığılı Plastik Davranış Modeli (Doğrusal olmayan yay)

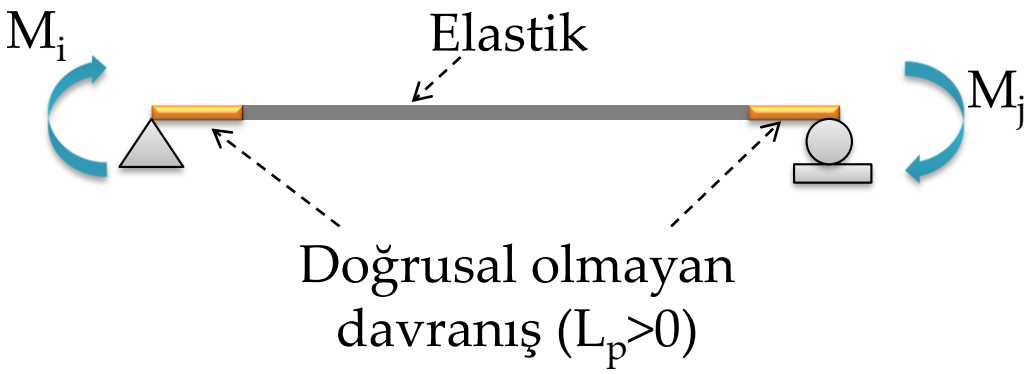


- Doğrusal olmayan davranışın yayda toplandığı varsayılır.
- Deney sonuçlarına bağlı fenemonolojik model.



Doğrusal Olmayan Davranış Modelleri

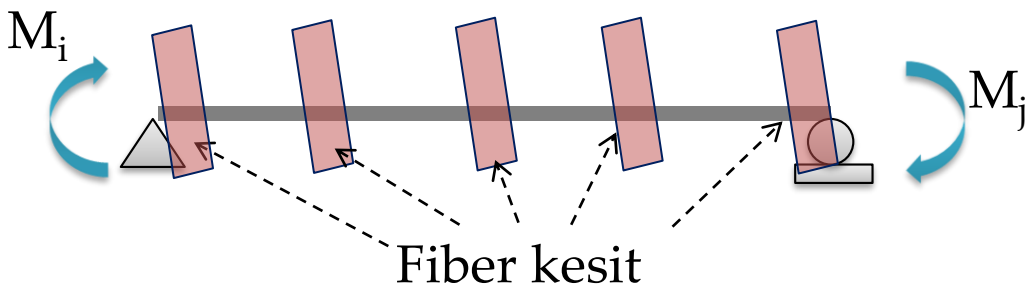
3. Yayılı Plastik Davranış Modeli (Sonlu uzunluklu mafsal modeli)



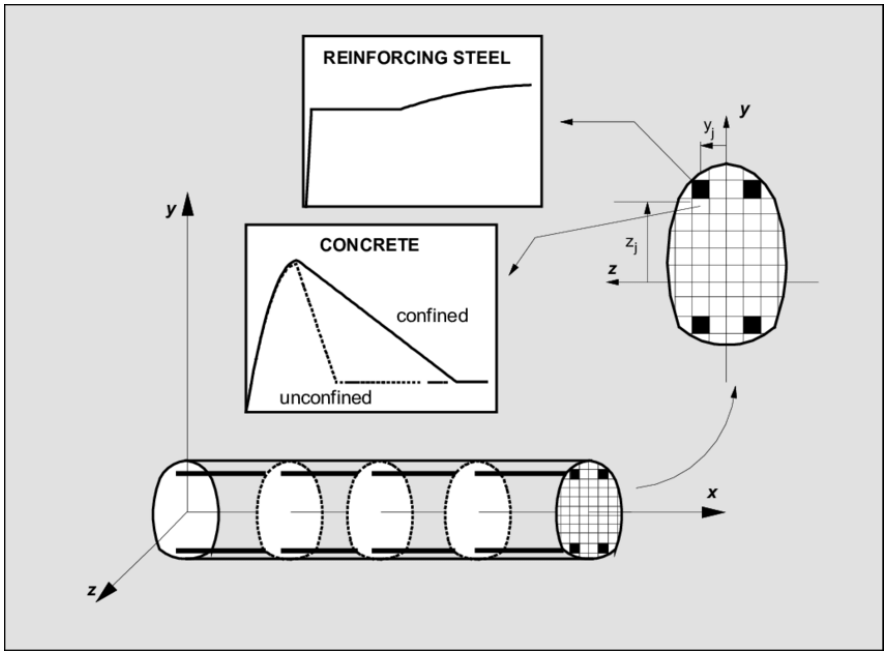
- Doğrusal olmayan şekil değiştirmeler sonlu plastik bölgelerde meydana gelir. Plastik bölgeye ait Moment-Eğrilik ilişkisi tanımlanabilir. Dönme, θ=Ortalama eğrilik×L_p olarak hesaplanabilir (L_p pratik olarak 0.5d alınabilir).
- Deney sonuçlarına bağlı fenemonolojik model.

Doğrusal Olmayan Davranış Modelleri

4. Yayılı Plastik Davranış Modeli (Lif modeli)

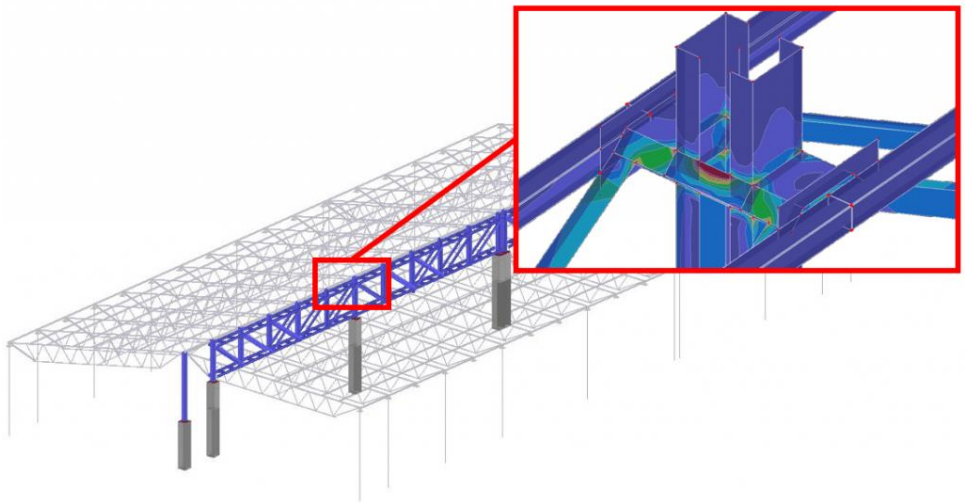


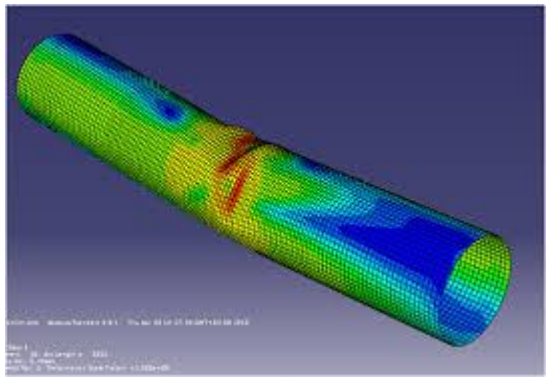
- Alternatif olarak, plastik bölgede, gerilme-birim şekildeğiştirme bağıntılarının çevrimsel olarak gözönüne alınabildiği kesit hücresi (lif) modeli.
- Deney sonucuna dayanmaz. Davranış liflere tanımlanan gerilme-şekildeğiştirme ilişkisinden elde edilir.

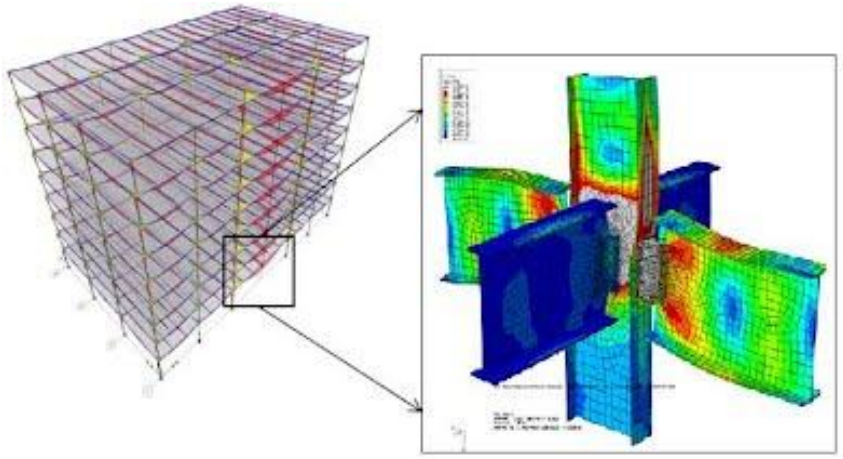


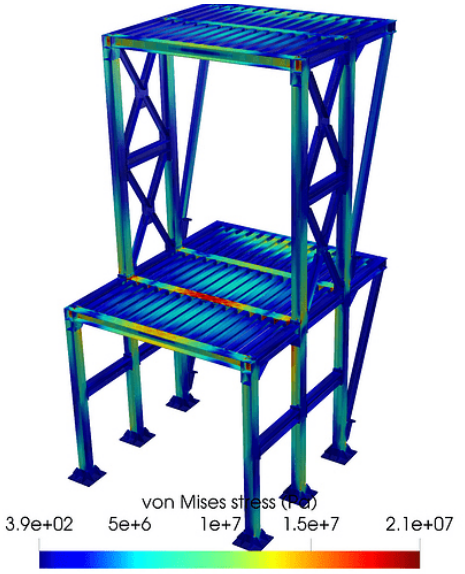
Doğrusal Olmayan Davranış Modelleri

5. Yayılı Plastik Davranış Modeli (Sonlu Eleman Modeli)

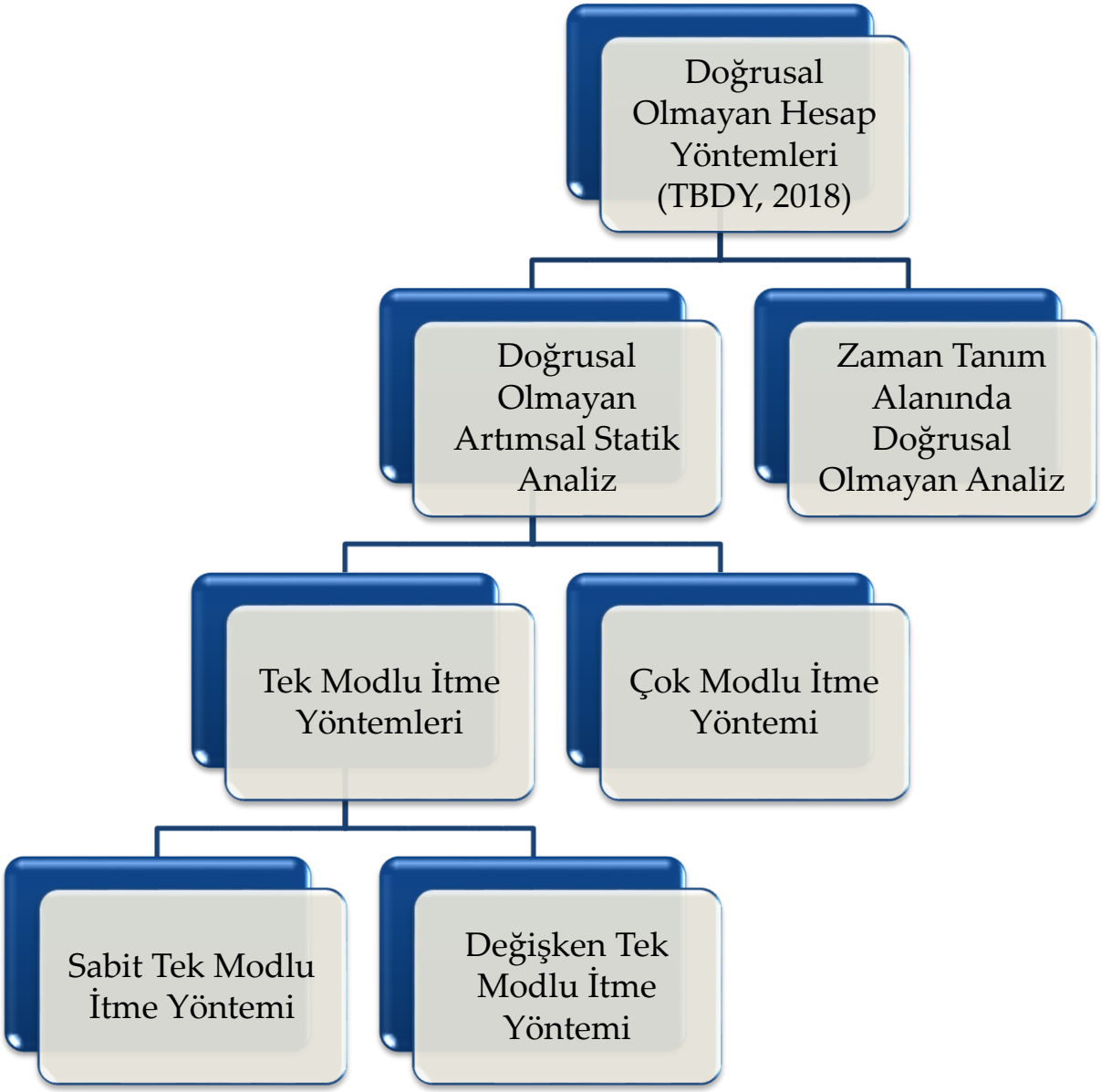








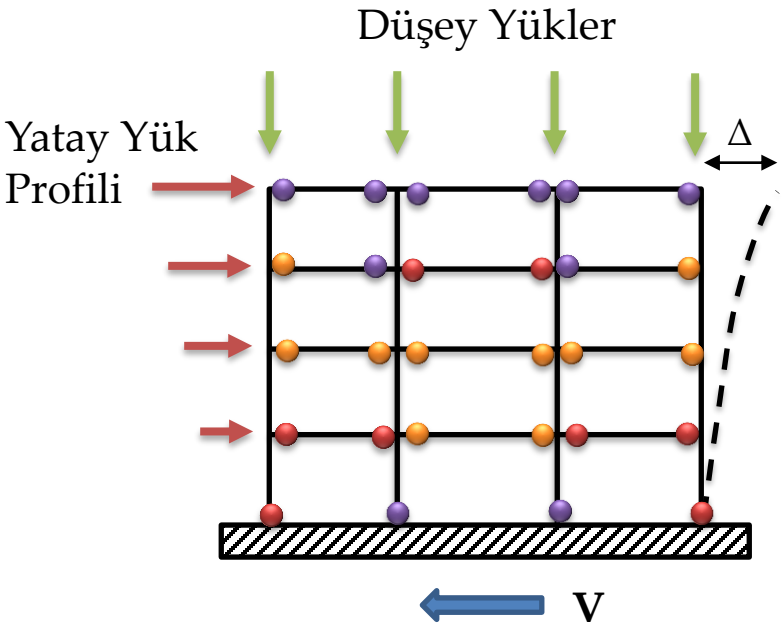
Doğrusal Olmayan Analiz Yöntemleri

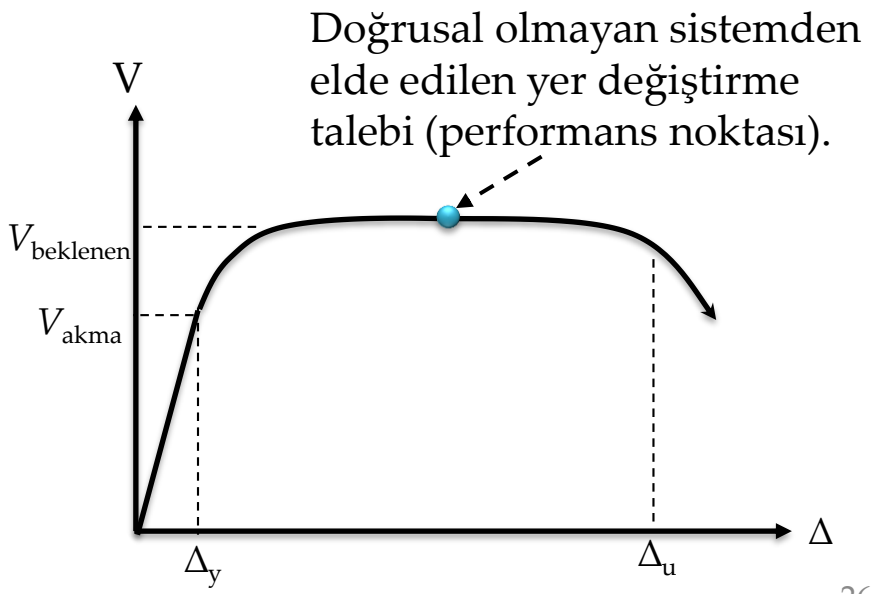


Artımsal Statik Analiz

Doğrusal Olmayan Artımsal Statik Analiz Adımları:

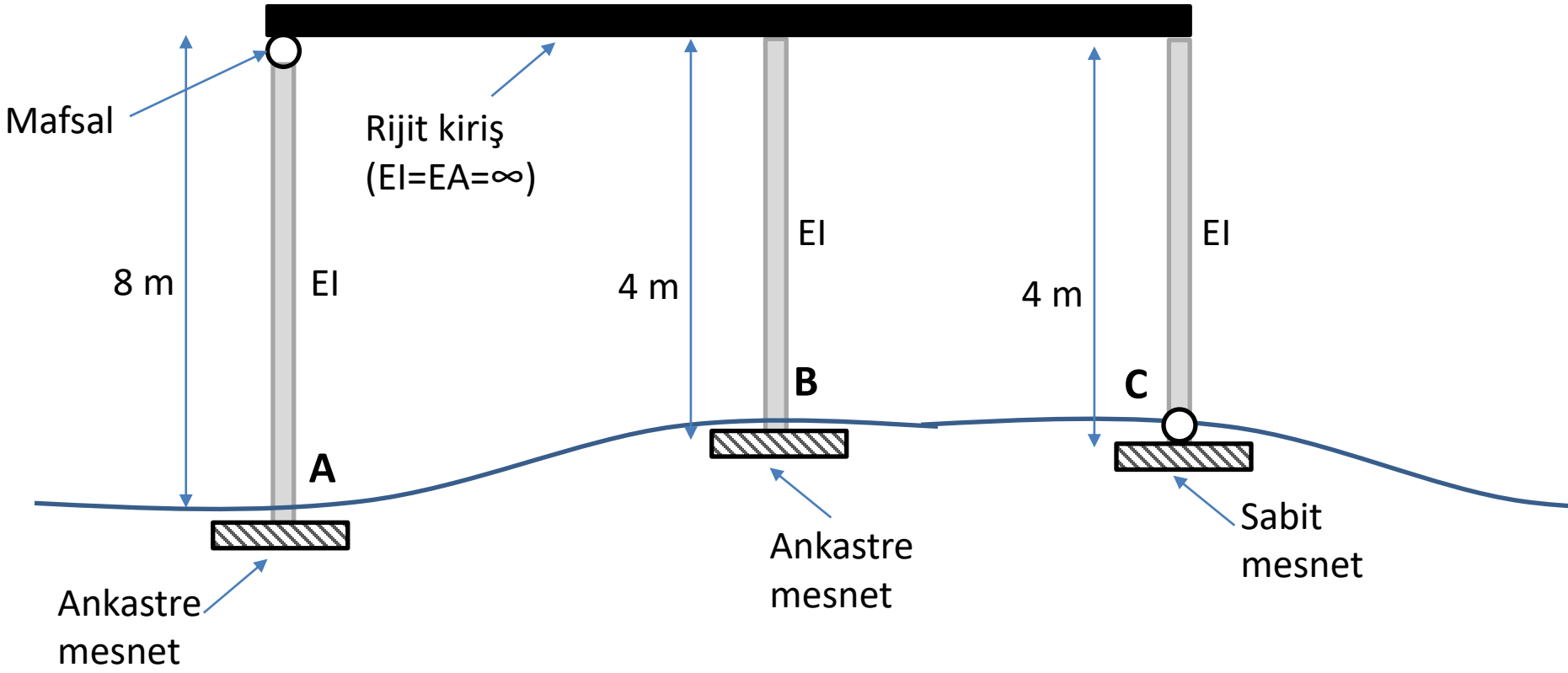
1. Düşey yüklerin uygulanması ve yatay yük profilinin belirlenmesi;
2. Kapasite eğrisinin elde edilmesi;
3. Elastik tepki spektrumu yardımıyla yer değiştirme talebinin tahmini;
4. Performans noktasında Talep/Kapasite oranlarının kontrolü.





Statik İtme Analizi (Elle Hesap)

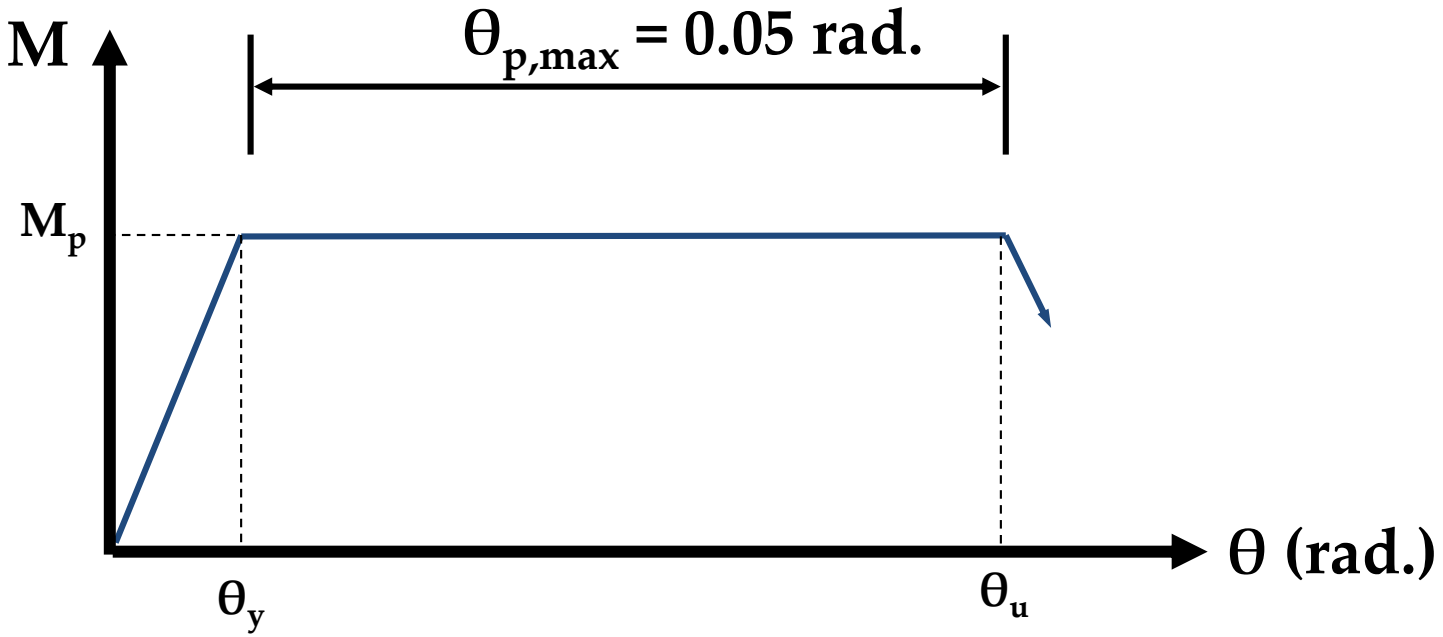
Statik İtme Analizi



Eleman	M_p (kN.m)	$\theta_{p,max}$ (rad)	EI (kN/m²)
Kolon	200	0.05	5000

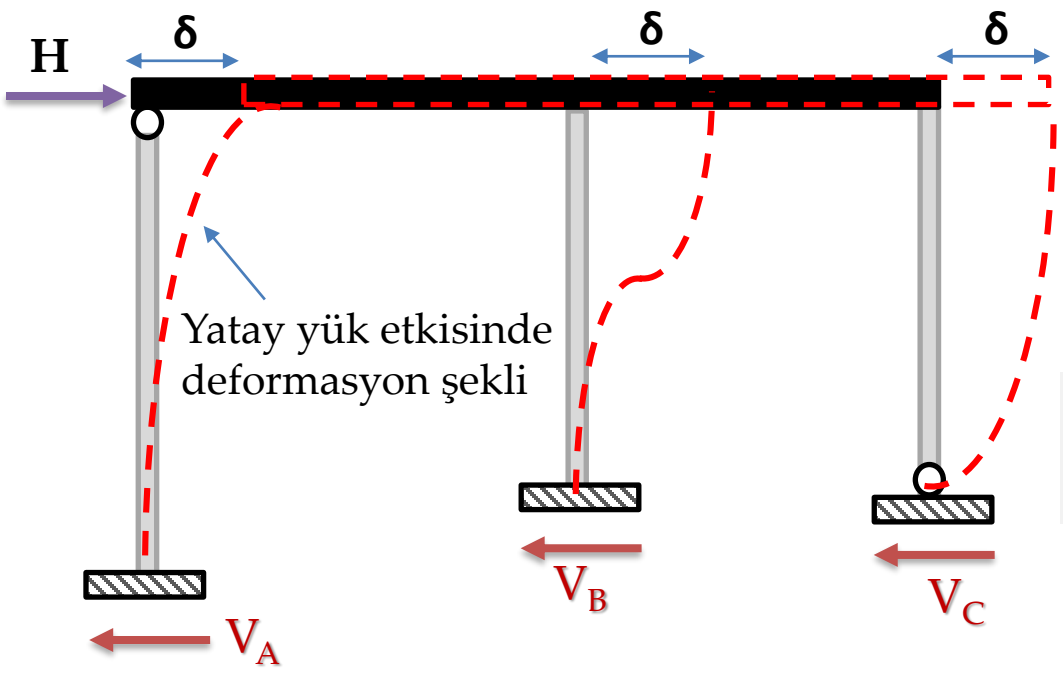
Statik İtme Analizi

Deney sonucu elde edilen moment-dönme ilişkisi.



Eleman	M_p (kN.m)	$\theta_{p,max}$ (rad)
Kolon	200	0.05

Statik İtme Analizi



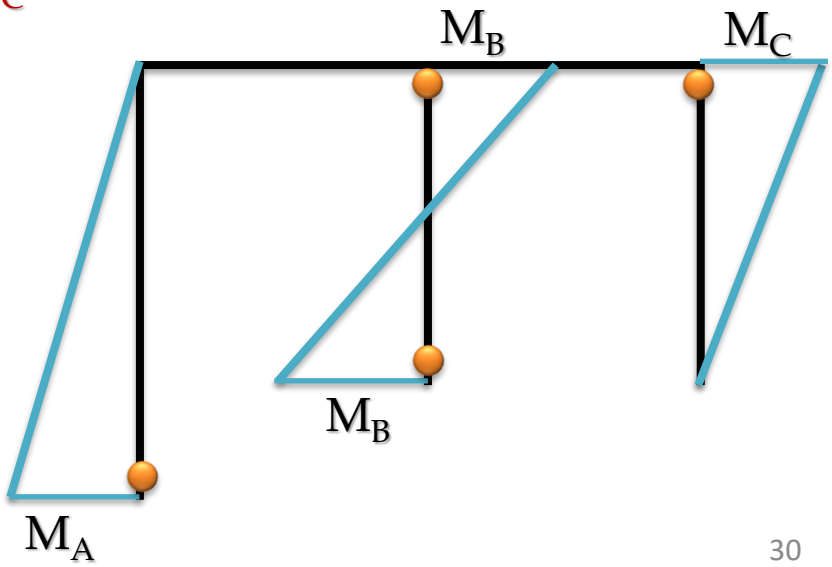
Toplam yatay yük, H, yatay rijitliklere oranlı olarak kolonlarda kesme kuvveti oluşturacaktır.

$$H = V_A + V_B + V_C$$

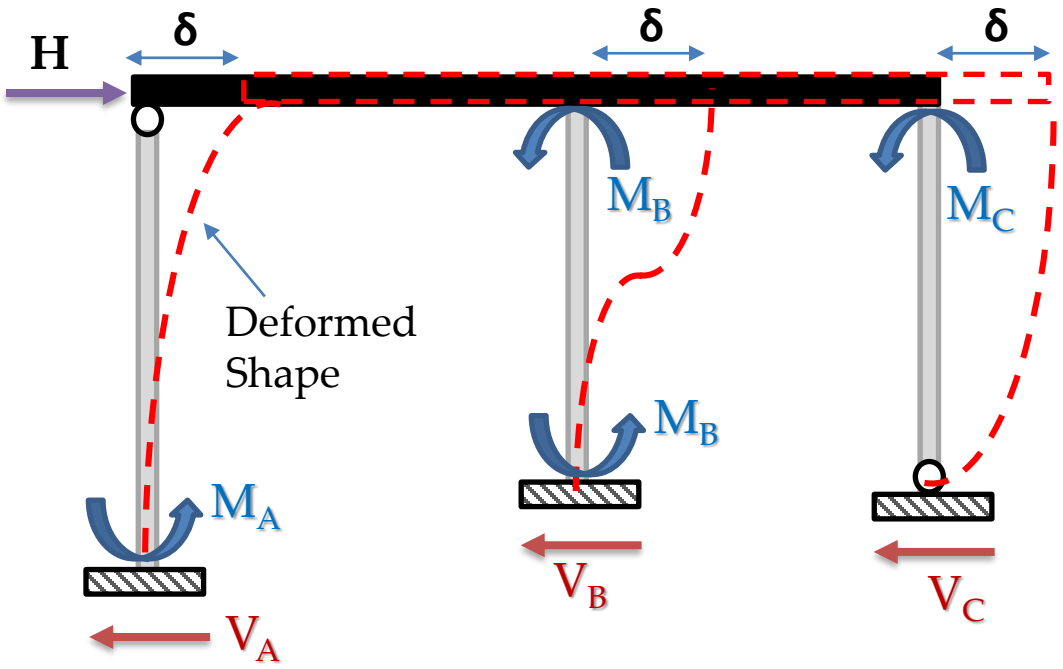
$$V_A = H\frac{k_A}{k_{01}}; V_B = H\frac{k_B}{k_{01}}; V_C = H\frac{k_C}{k_{01}}$$

Rijitlik Katsayıları

$$\left.\begin{aligned} k_A &= \frac{3EI}{L_A^{\,3}} = \frac{3EI}{512} \\ k_B &= \frac{12EI}{L_B^{\,3}} = \frac{12EI}{64} \\ k_C &= \frac{3EI}{L_C^{\,3}} = \frac{3EI}{64} \end{aligned}\right\} k_{01} = k_A + k_B + k_C = \frac{123EI}{512}$$



Statik İtme Analizi



$$V_A = H\frac{k_A}{k_{01}} = \frac{3H}{123}$$

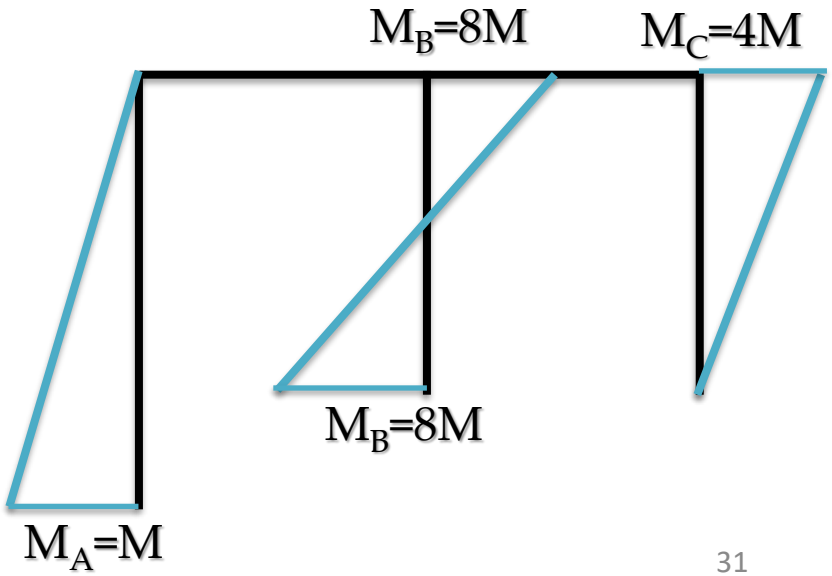
$$V_B = H\frac{k_B}{k_{01}} = \frac{96H}{123}$$

$$V_C = H\frac{k_C}{k_{01}} = \frac{24H}{123}$$

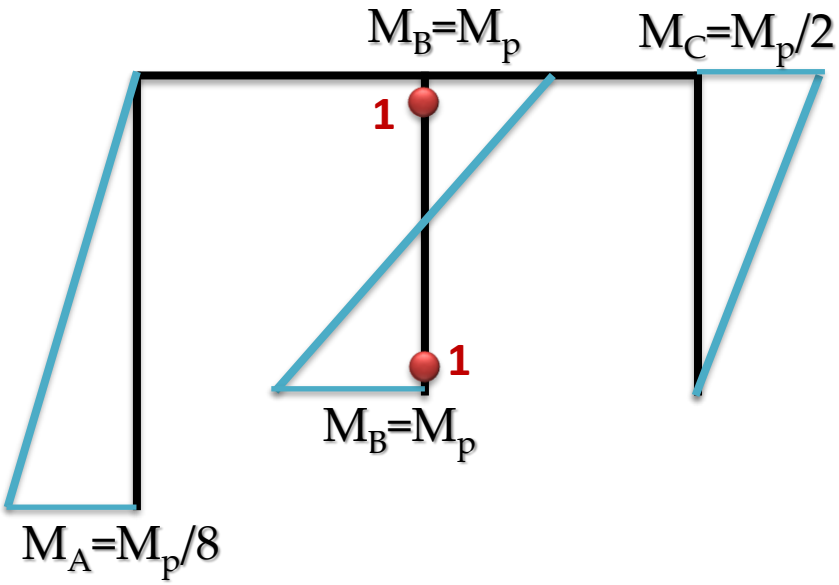
$$V_A = \frac{M_A}{L_A} \rightarrow M_A = 8V_A = 8\times\left(\frac{3H_y}{123}\right) = \frac{24H_y}{123}$$

$$V_B = \frac{2M_B}{L_B} \rightarrow M_B = 2V_B = 2\times\left(\frac{96H_y}{123}\right) = \frac{192H_y}{123}$$

$$V_C = \frac{M_C}{L_C} \rightarrow M_C = 4V_C = 4\times\left(\frac{24H_y}{123}\right) = \frac{96H_y}{123}$$



Statik İtme Analizi



$$M_A = \frac{24H_y}{123}$$

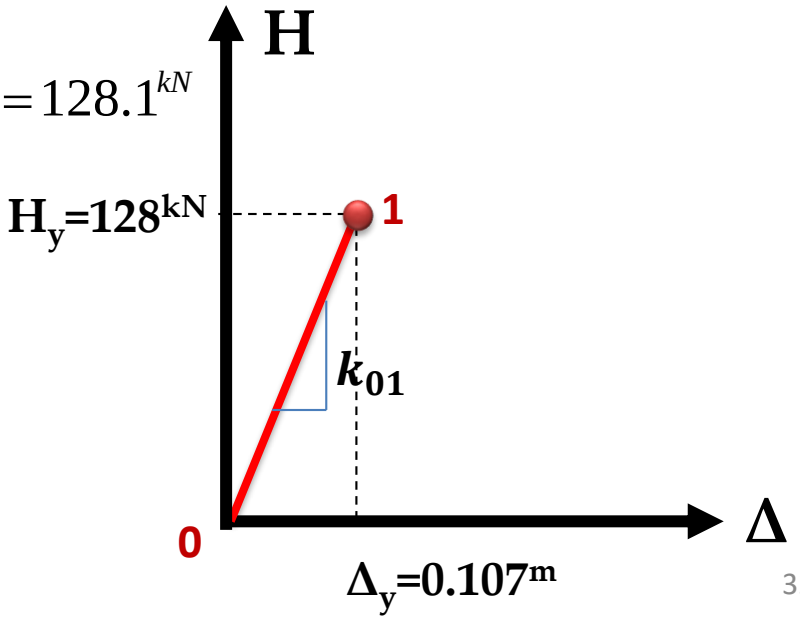
$$M_B = \frac{192H_y}{123}$$

$$M_C = \frac{96H_y}{123}$$

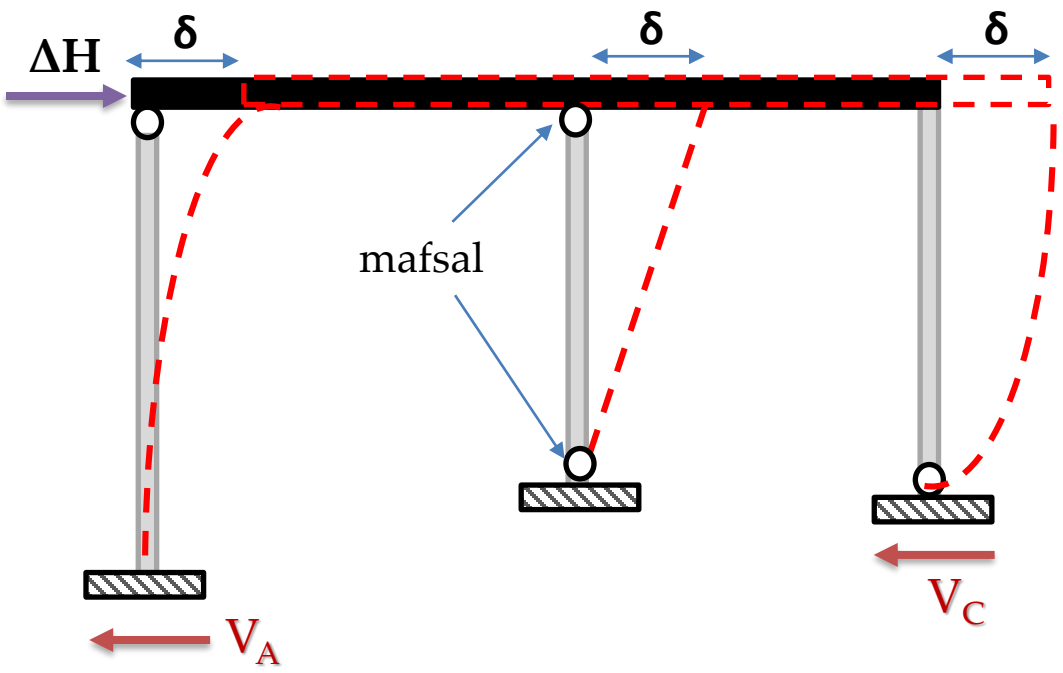
İlk olarak, Kolon B plastik kapasitesine (M_p) ulaşır.

$$M_p = \frac{192H_y}{123} \rightarrow H_y = \frac{192M_p}{123} = \frac{192\times 200}{123} = 128.1^{kN}$$

$$\Delta_y = \frac{H_y}{k_{01}} = \frac{128.1^{kN}}{123EI/512} = 0.107^{m}$$



Statik İtme Analizi



Plastik mafsal oluşumundan sonra yapı modifiye edildi.

$$\Delta H = V_A + V_C$$

$$V_A = \Delta H \frac{k_A}{k_{12}}; V_C = \Delta H \frac{k_C}{k_{12}}$$

$$V_A = \frac{\Delta H}{9}; V_C = \frac{8\Delta H}{9}$$

1. Adımdan sonra kalan eğilme momenti kapasiteleri

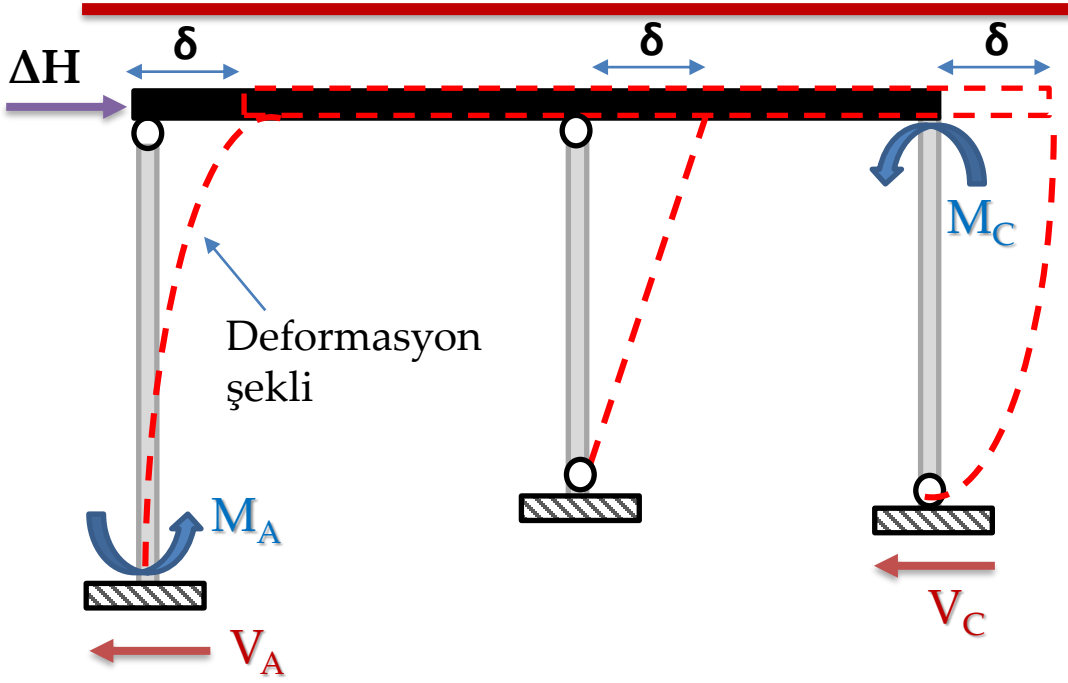
$$M_A = M_p - M_p/8 = 7M_p/8$$

$$M_C = M_p - M_p/2 = M_p/2$$

Yatay rijitlikler

$$\left.\begin{aligned} k_A &= \frac{3EI}{L_A^{\,3}} = \frac{3EI}{512} \\ k_C &= \frac{3EI}{L_C^{\,3}} = \frac{24EI}{512} \end{aligned}\right\} k_{12} = k_A + k_C = \frac{27EI}{512}$$

Statik İtme Analizi



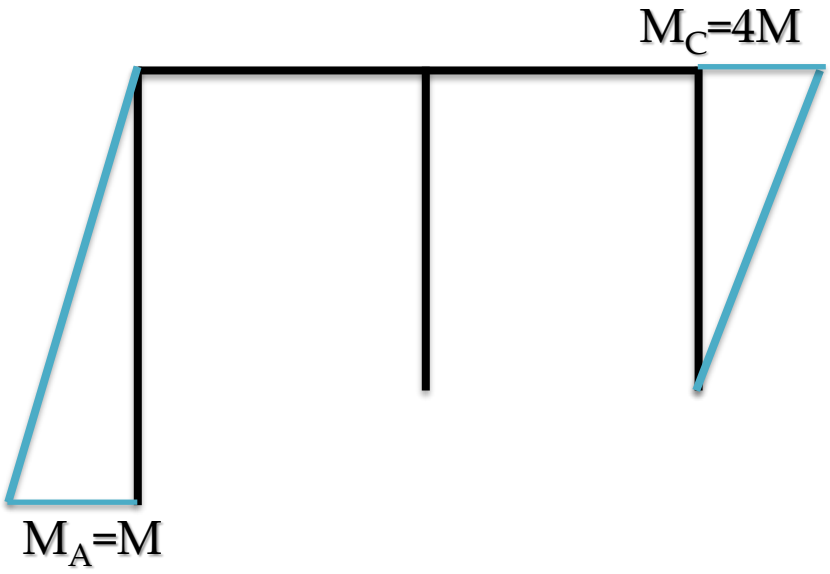
$$Kolon\ A \rightarrow M_A = 7M_p / 8$$

$$Kolon\ C \rightarrow M_C = M_p / 2$$

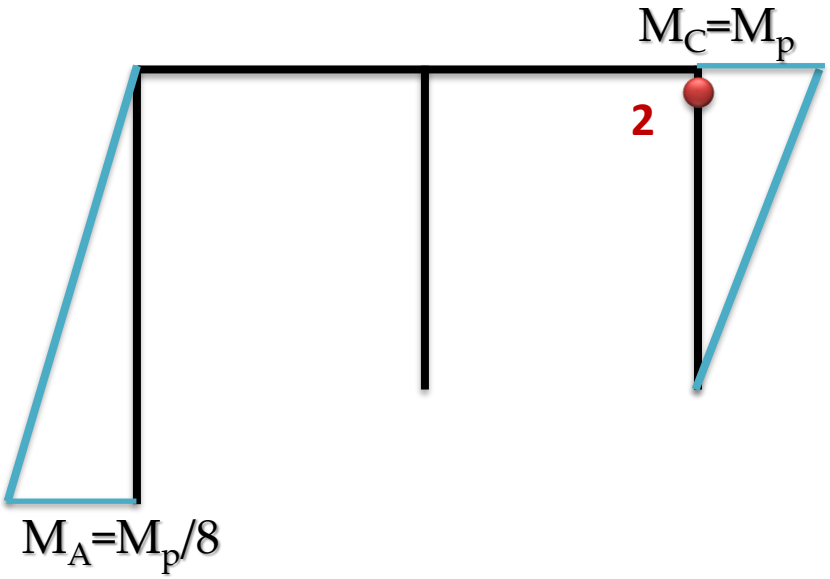
$$V_A = \frac{\Delta H}{9}; V_C = \frac{8\Delta H}{9}$$

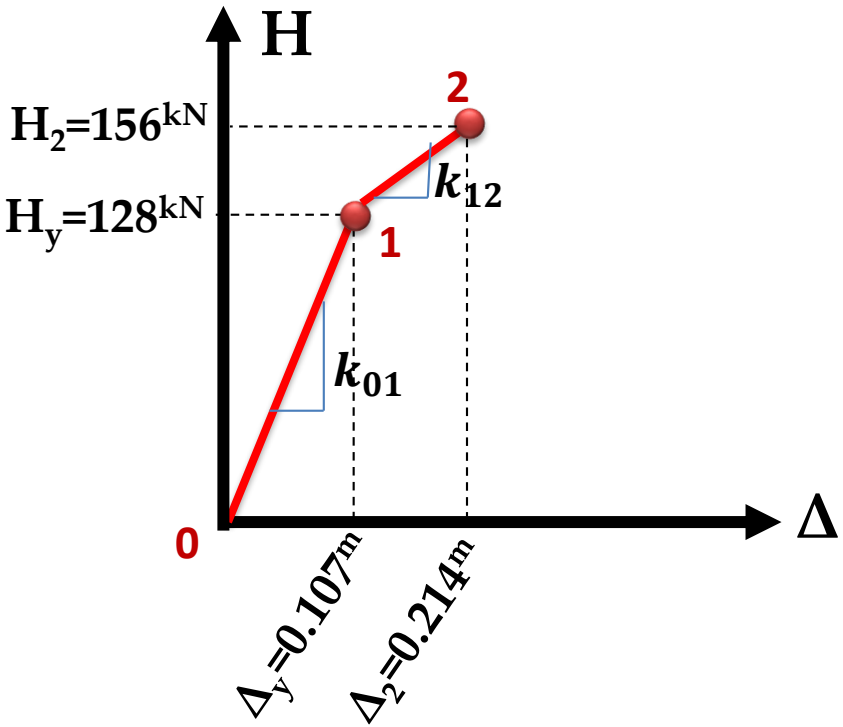
$$M_A = 8V_A = 8 \times \left(\frac{\Delta H}{9} \right) = \frac{8\Delta H}{9}$$

$$M_C = 4V_C = 4 \times \left(\frac{8\Delta H}{9} \right) = \frac{32\Delta H}{9}$$



Statik İtme Analizi





Kolon A

$$7M_p/8=\frac{8\Delta H}{9}\rightarrow \Delta H=\frac{72M_p}{64}$$

$$\Delta H=\frac{9M_p}{64}=\frac{9\times 200}{64}=28.1^{kN}$$

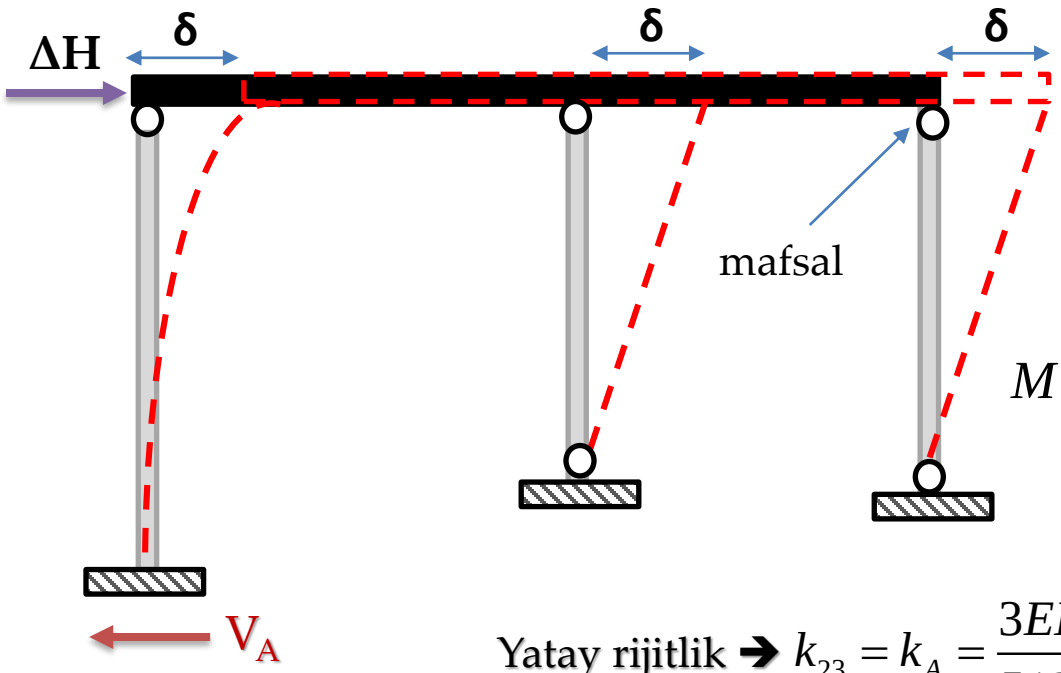
Kolon C

$$M_p/2=\frac{32\Delta H}{9}\rightarrow \Delta H=\frac{9M_p}{64}$$

Kolon C M_p kapasitesine ulaşır.

$$\Delta_2-\Delta_y=\frac{\Delta H}{k_{12}}=\frac{28.1^{kN}}{27EI/512}=0.107^m$$

Statik İtme Analizi



Plastik mafsal oluşumundan sonra yapı modifiye edildi.

$$\Delta H = V_A$$

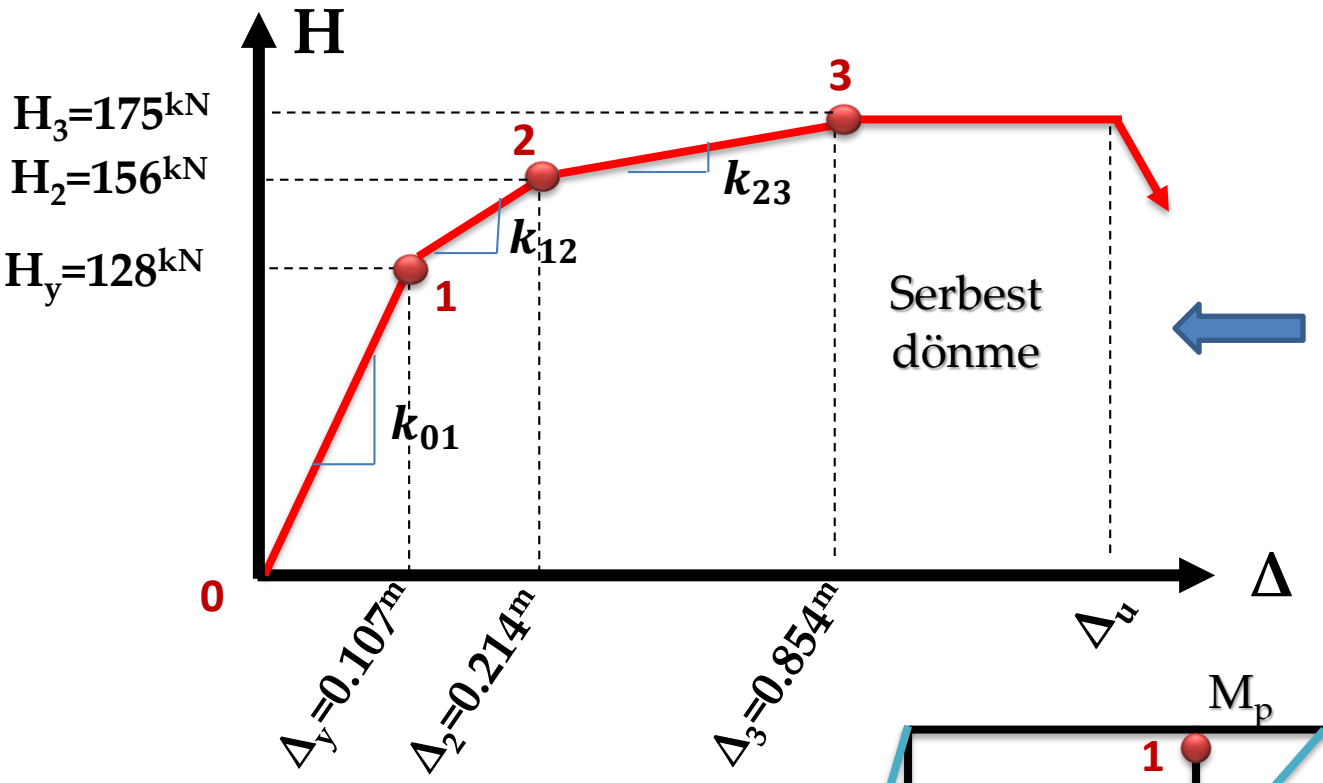
Kalan moment kapasitesi

$$M_A = M_p - M_p/8 - M_p/8 = 3M_p/4$$

Yatay rijitlik ➔ $k_{23} = k_A = \dfrac{3EI}{512}$

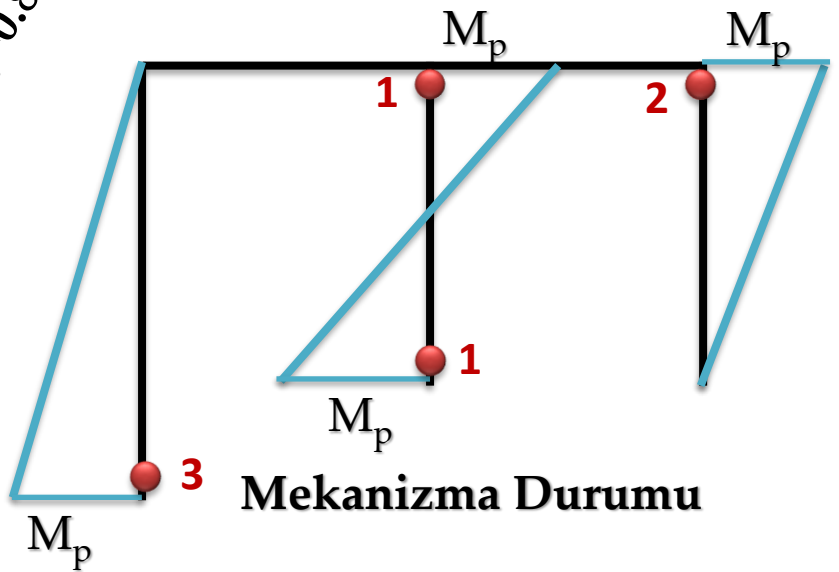
$$\Delta H \times L_A = \frac{3M_p}{4} \Rightarrow \Delta H = \frac{3\times 200}{32} = 18.8^{kN} \qquad \Delta_3 - \Delta_2 = \frac{\Delta H}{k_{23}} = \frac{18.8^{kN}}{3EI/512} = 0.64^{m}$$

Statik İtme Analizi



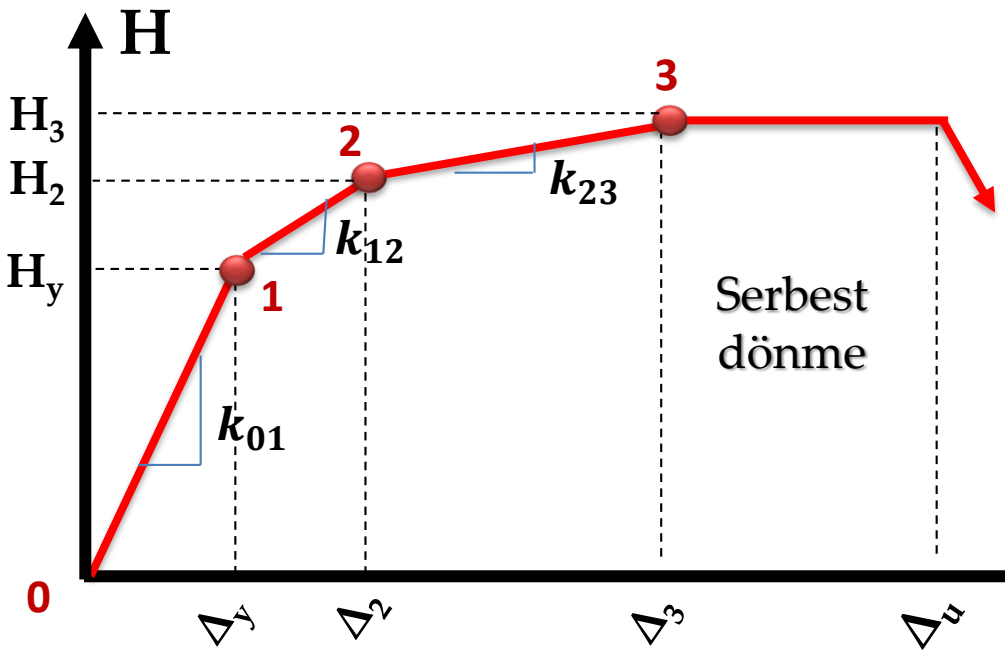
Kolonların yeterli dönme kapasitesine sahip olduğu varsayıldı!

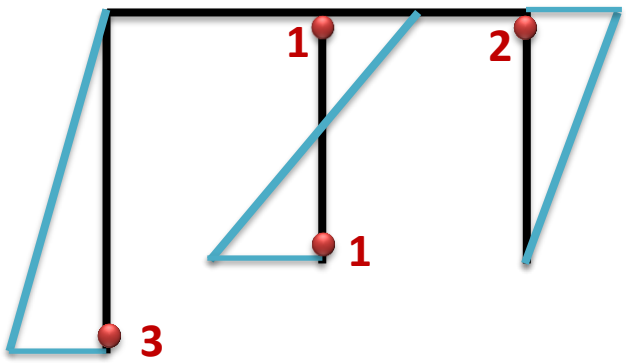
Kolonların plastik dönme kapasiteleri kontrol edilmeli!



Mekanizma Durumu

Statik İtme Analizi





Kolon B için plastik bölge

$$\Delta_p^B = \Delta_u - \Delta_y$$

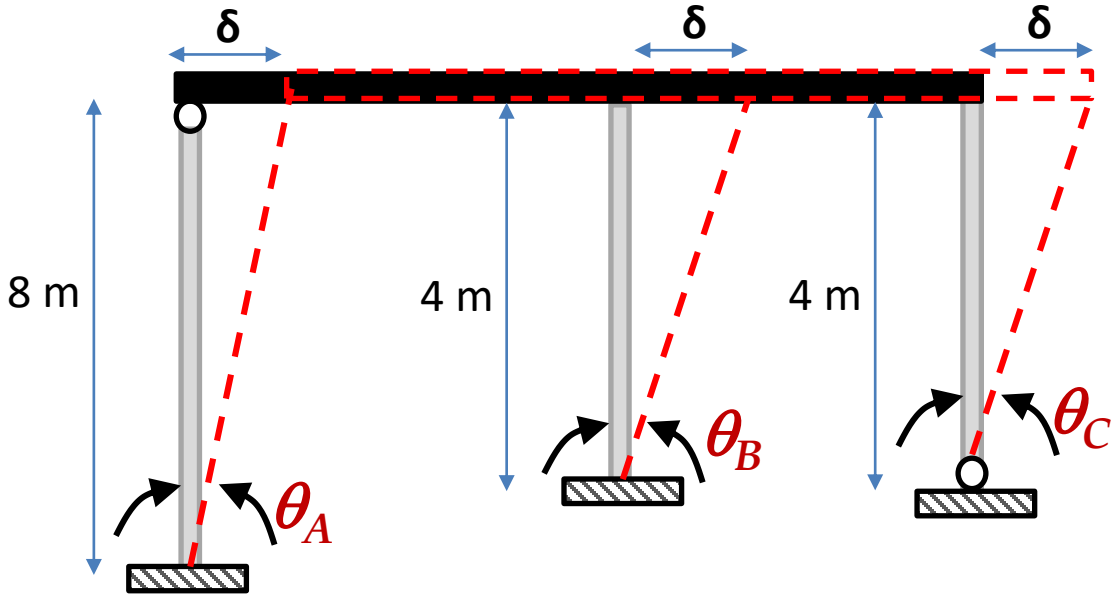
Kolon C için plastik bölge

$$\Delta_p^C = \Delta_u - \Delta_2$$

Kolon A için plastik bölge

$$\Delta_p^A = \Delta_u - \Delta_3$$

Statik İtme Analizi



$$\Delta_y = 0.107^m$$

$$\Delta_2 = 0.214^m$$

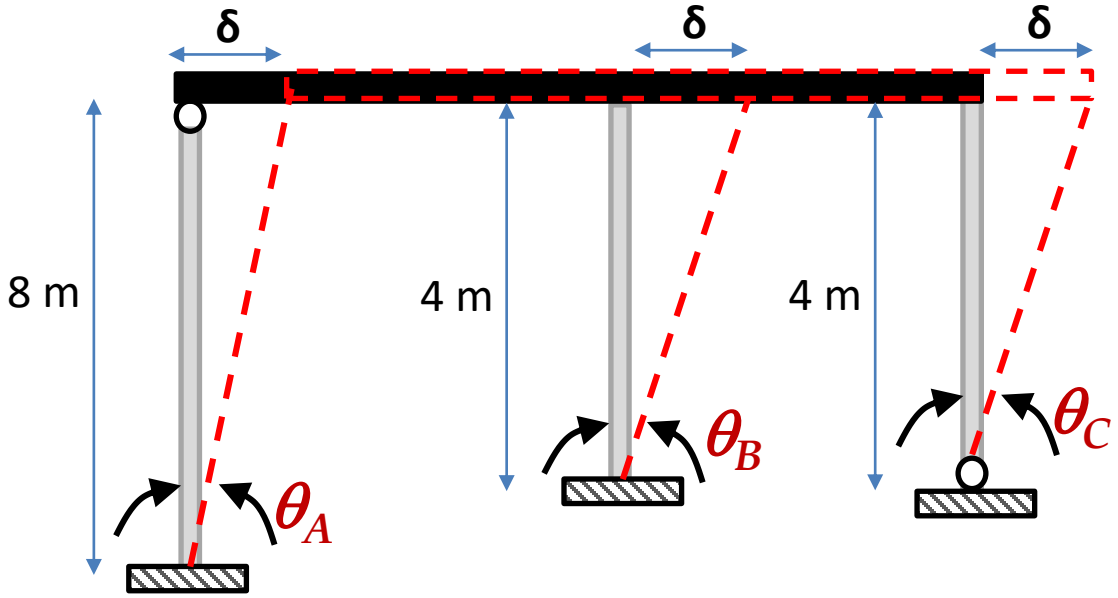
$$\Delta_3 = 0.854^m$$

$$\theta_p{}^B = \frac{\Delta_p^B}{L_B} = \frac{\Delta_u - \Delta_y}{L_B} \le \theta_{p,\max} = 0.05 \Rightarrow \Delta_u = \theta_{p,\max} L_B + \Delta_y$$

$$\theta_p{}^C = \frac{\Delta_p^C}{L_C} = \frac{\Delta_u - \Delta_2}{L_C} \le \theta_{p,\max} = 0.05 \Rightarrow \Delta_u = \theta_{p,\max} L_C + \Delta_2$$

$$\theta_p{}^A = \frac{\Delta_p^A}{L_A} = \frac{\Delta_u - \Delta_3}{L_A} \le \theta_{p,\max} = 0.05 \Rightarrow \Delta_u = \theta_{p,\max} L_A + \Delta_3$$

Statik İtme Analizi



$$\Delta_y = 0.107^m$$

$$\Delta_2 = 0.214^m$$

$$\Delta_3 = 0.854^m$$

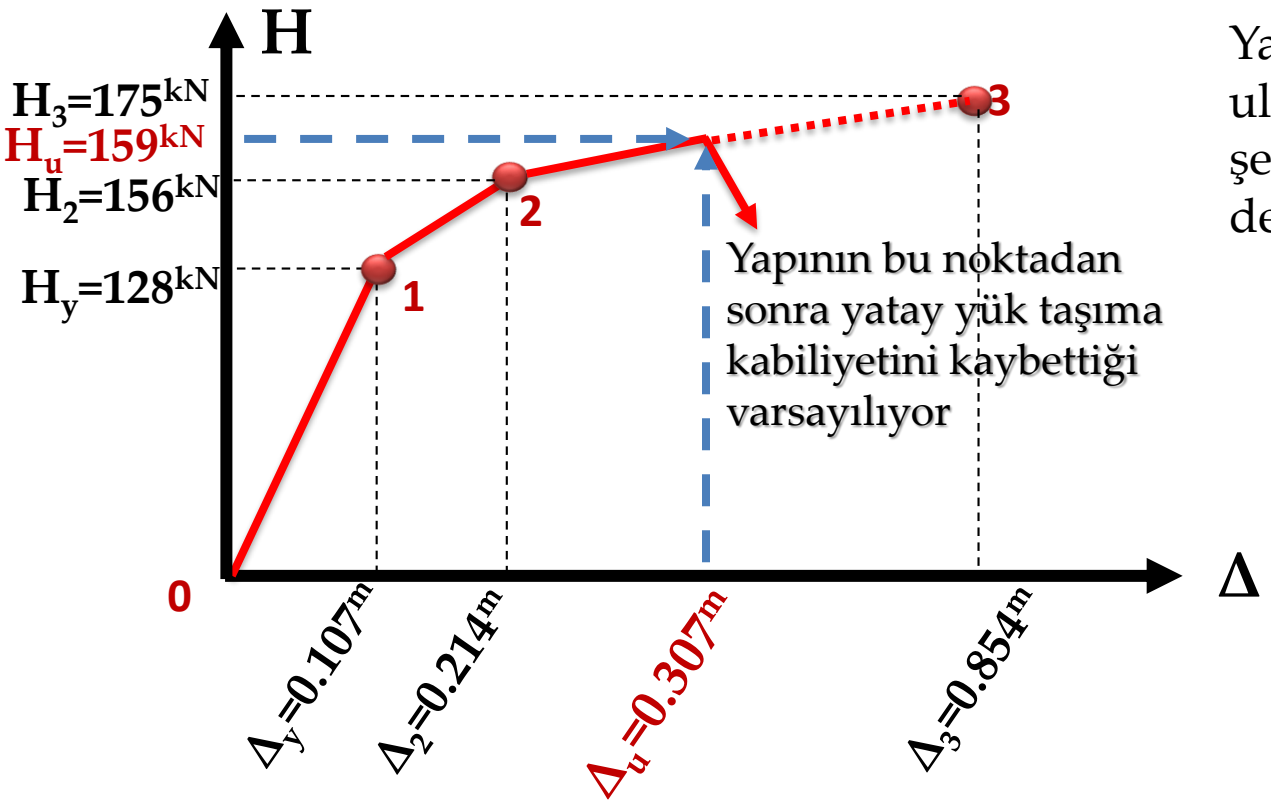
$$\Delta_u = \theta_{p,\max} L_B + \Delta_y \rightarrow \Delta_u = 0.05 \times 4^m + 0.107^m = 0.307^m$$

$$\Delta_u = \theta_{p,\max} L_C + \Delta_2 \rightarrow \Delta_u = 0.05 \times 4^m + 0.214^m = 0.414^m$$

$$\Delta_u = \theta_{p,\max} L_A + \Delta_3 \rightarrow \Delta_u = 0.05 \times 8^m + 0.854^m = 1.254^m$$

Kolon B yatay yerdeğiştirme 0.307^m iken plastik dönme kapasitesine ulaşıyor.

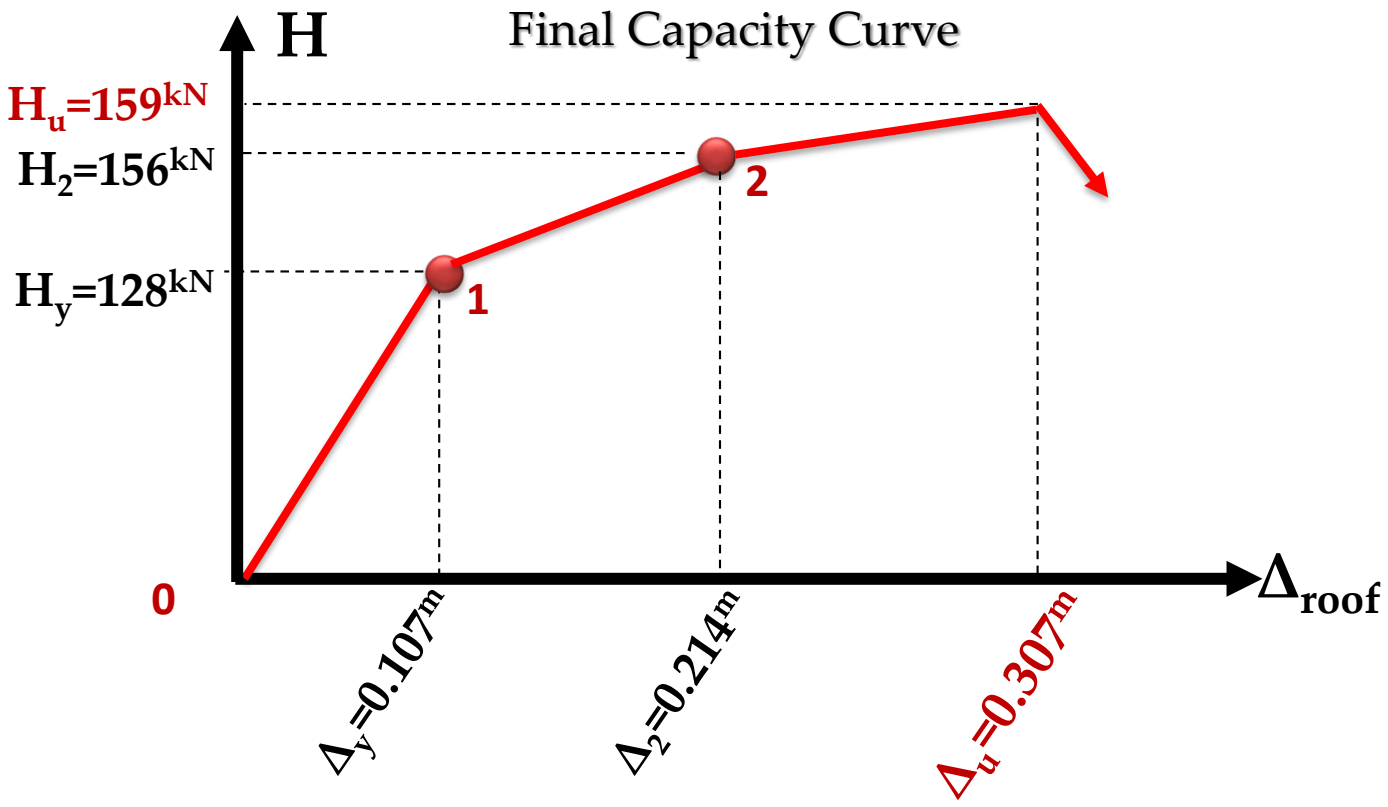
Statik İtme Analizi



Yapı, 3 noktasına ulaşmadan şekildeğiştime sınır değerine ulaşıyor.

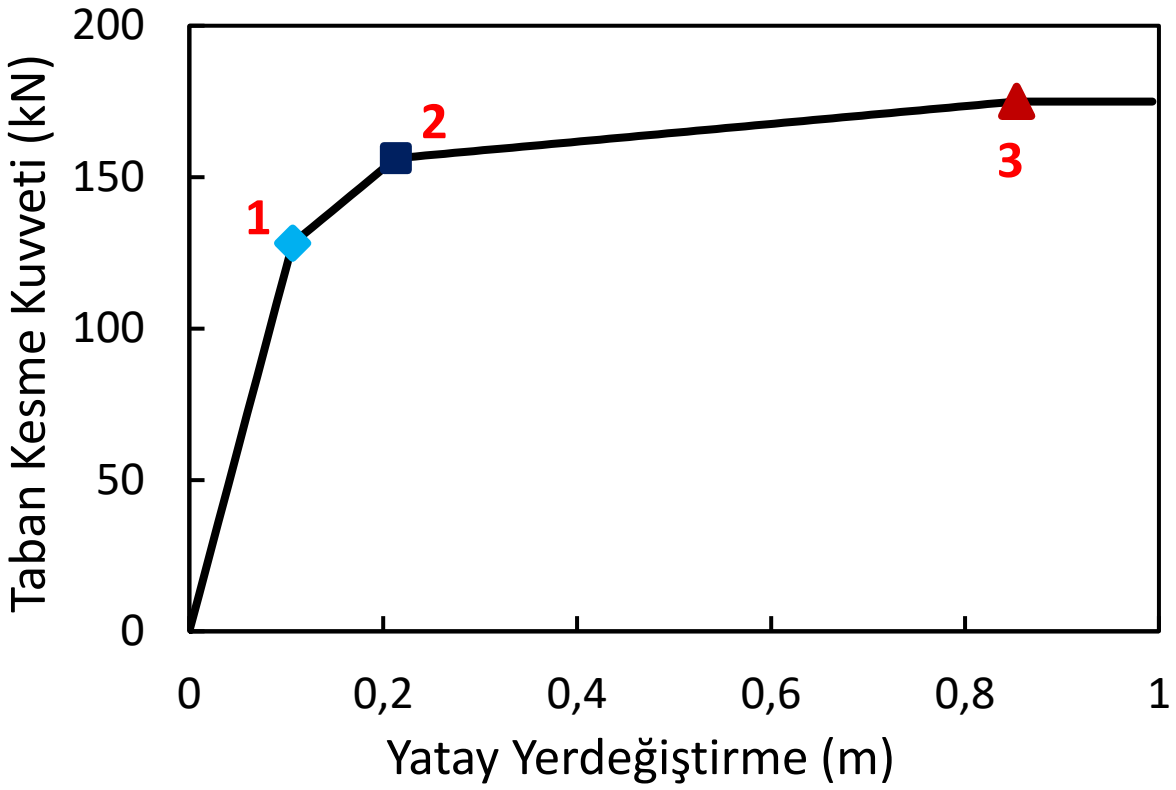
$$H_u = H_2 + (H_3 - H_2)\frac{\Delta_u - \Delta_2}{\Delta_3 - \Delta_2} = 156 + (175 - 156) \times \frac{0.307 - 0.214}{0.854 - 0.214} = 159^{kN}$$

Statik İtme Analizi



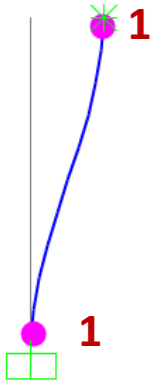
$$Sistem\ Sünekliği = \mu = \frac{\Delta_u}{\Delta_y} = \frac{0.307}{0.107} = 2.87$$

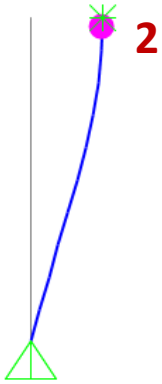
Statik İtme Analizi (Bilgisayar Destekli)

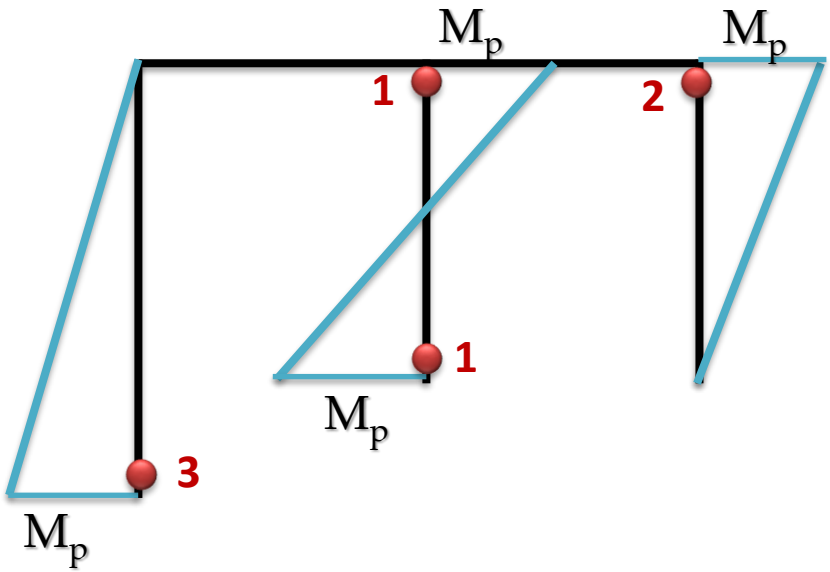


Statik İtme Analizi

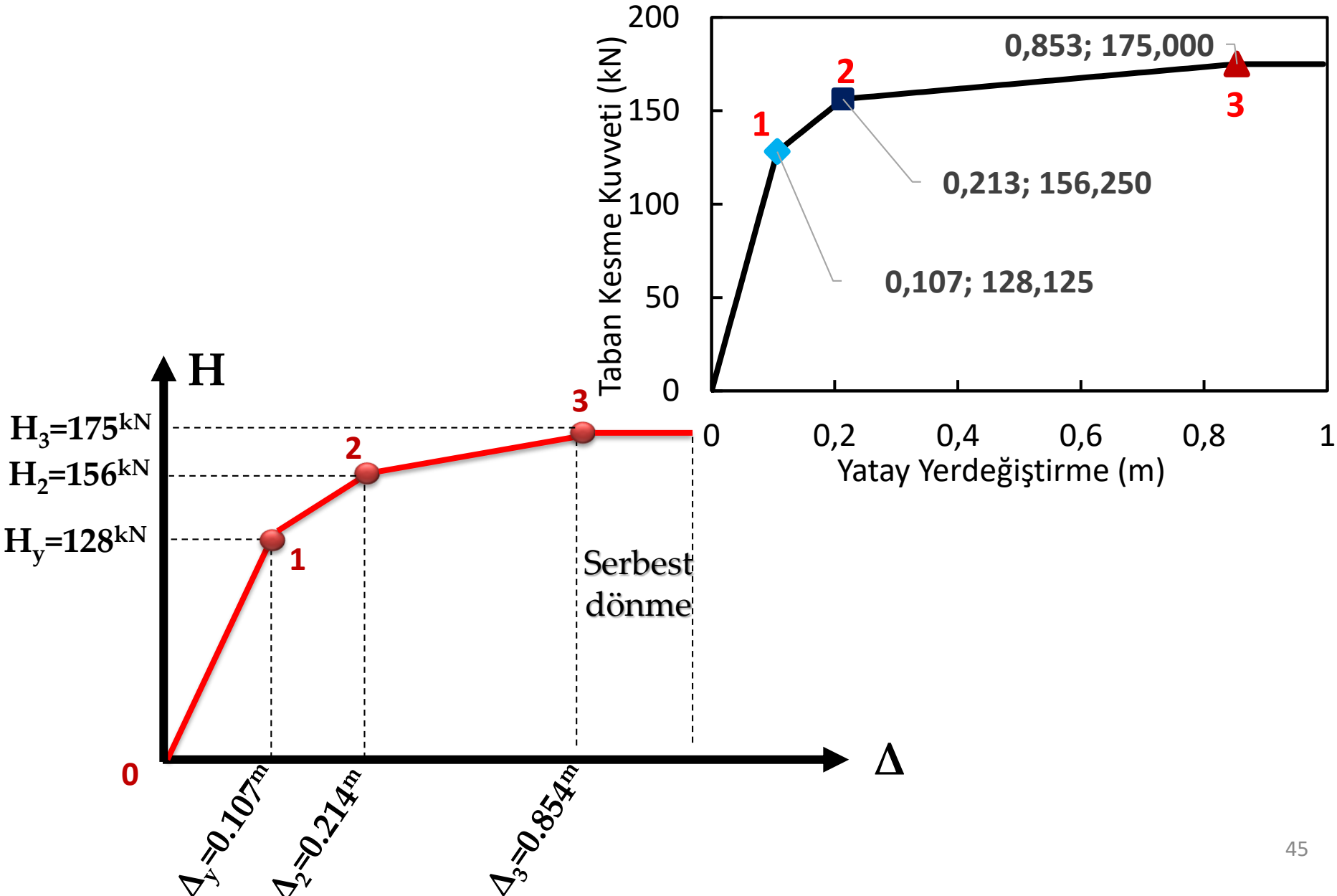








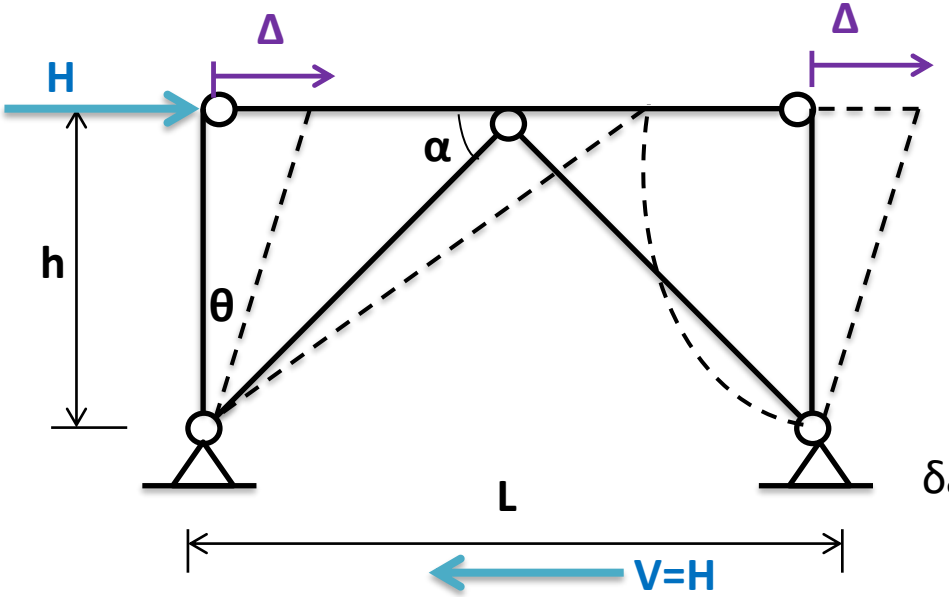
Statik İtme Analizi

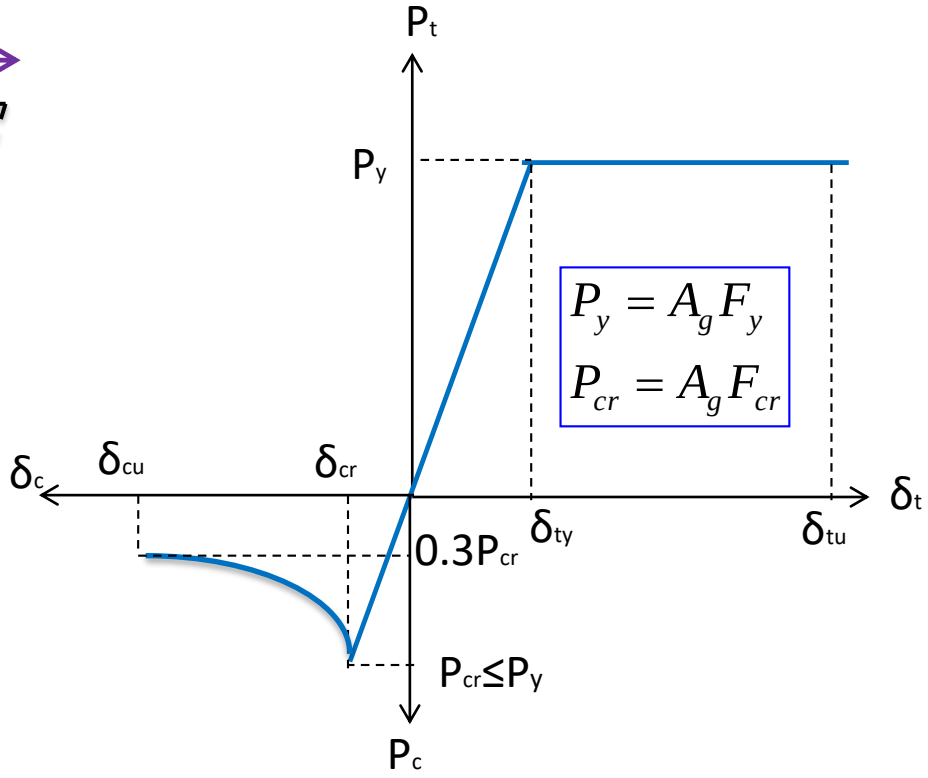


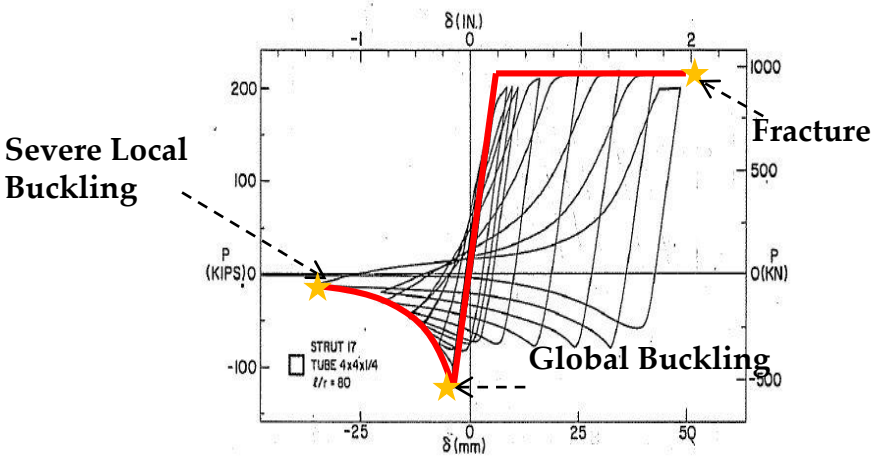
Statik İtme Analizi
(Merkezi Çaprazlı Çerçeve)

Inelastik Analiz

Inelastic Static Analysis of One-Story One-Bay CBF

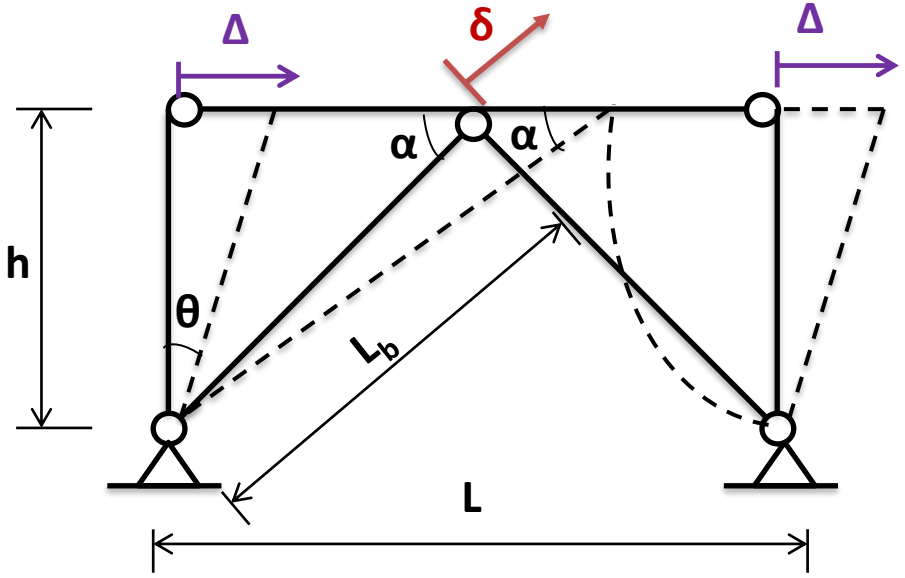






Kinematik İlişkiler

Kinematic relationships between axial displacement and interstory drift



In this relationship,

θ is the interstory drift angle (i.e., story drift ratio),

δ denotes the axial displacement corresponding to a drift angle,

L_b is the end-to-end brace length,

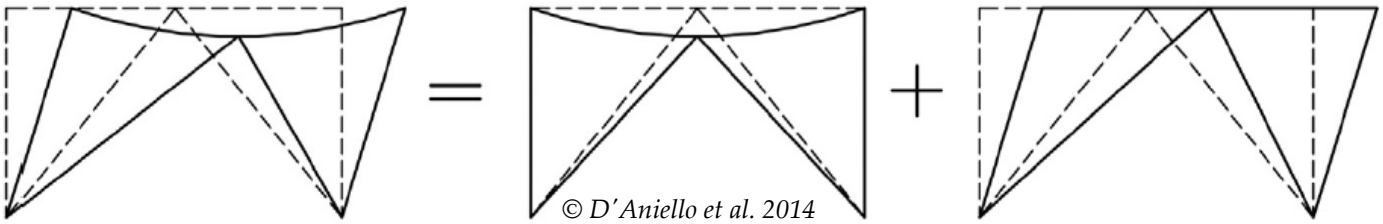
L is the span length, and

α represents the angle between the bracing and the girder,

$$\text{Story Drift Ratio (SDR)} \Rightarrow \theta = \frac{\Delta}{h}$$

$$\delta = \Delta \times \cos\alpha \qquad h = L_b \times \sin\alpha$$

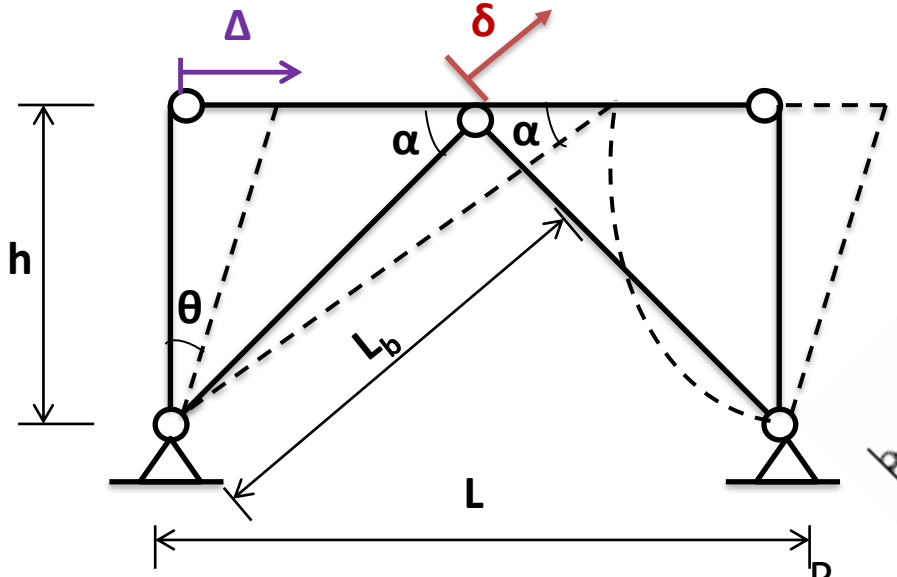
$$\frac{\delta}{L_b} = \theta \sin\alpha \cos\alpha$$

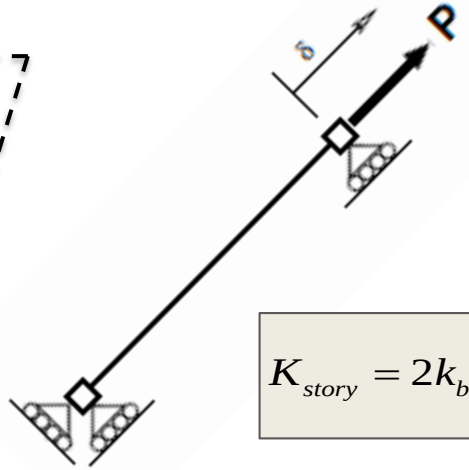


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Eksenel-Yatay Rijitlik İlişkisi

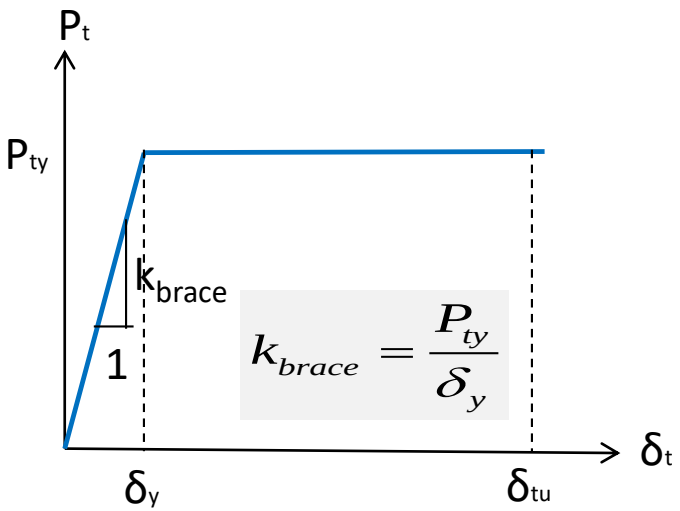
Axial stiffness vs. Lateral stiffness Relationship (Elastic)



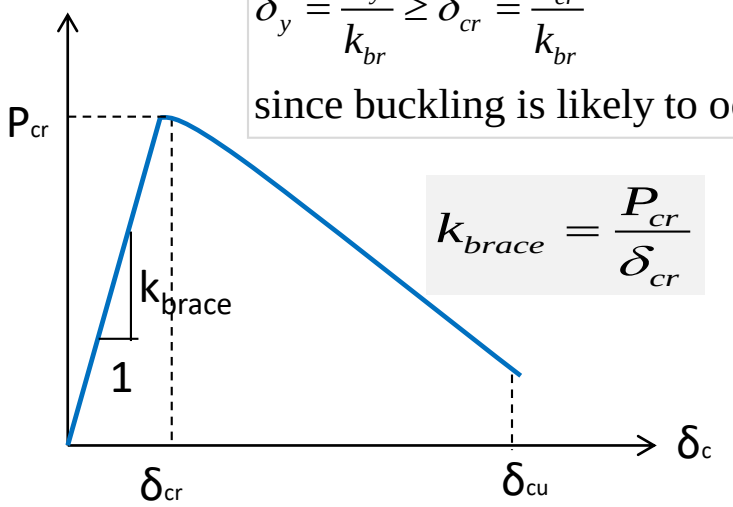


$$k_{brace} = \frac{EA}{L_b}$$

$$K_{story} = 2k_{brace}\cos^2\alpha = 2\left(\frac{EA}{L_b}\right)\cos^2\alpha$$



$$k_{brace} = \frac{P_{ty}}{\delta_y}$$

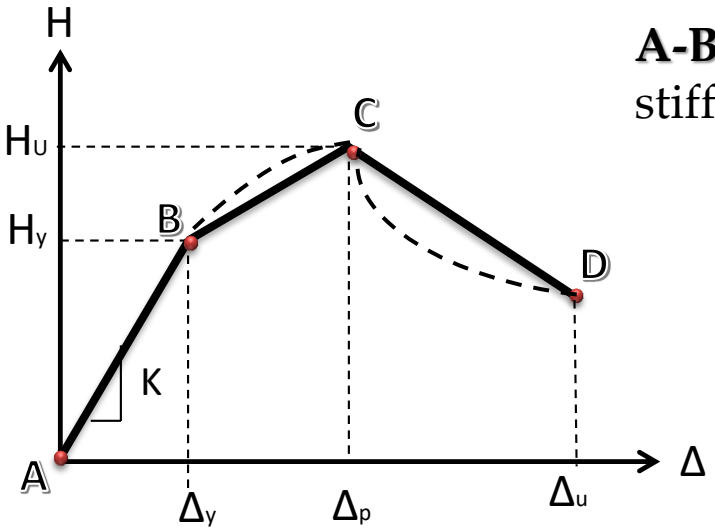


$$\delta_y = \frac{P_y}{k_{br}} \geq \delta_{cr} = \frac{P_{cr}}{k_{br}}$$

since buckling is likely to occur before yielding!

$$k_{brace} = \frac{P_{cr}}{\delta_{cr}}$$

Inelastik Analiz

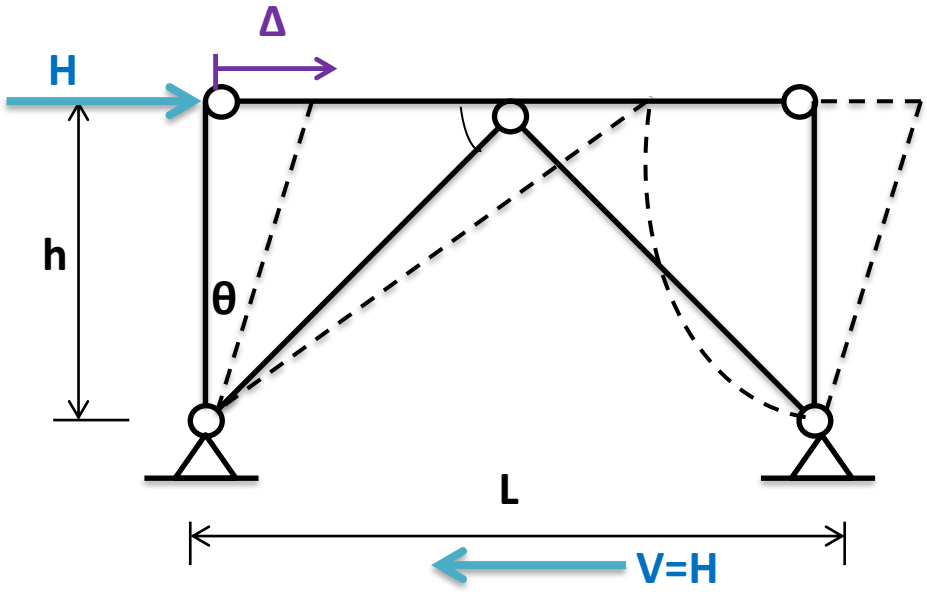


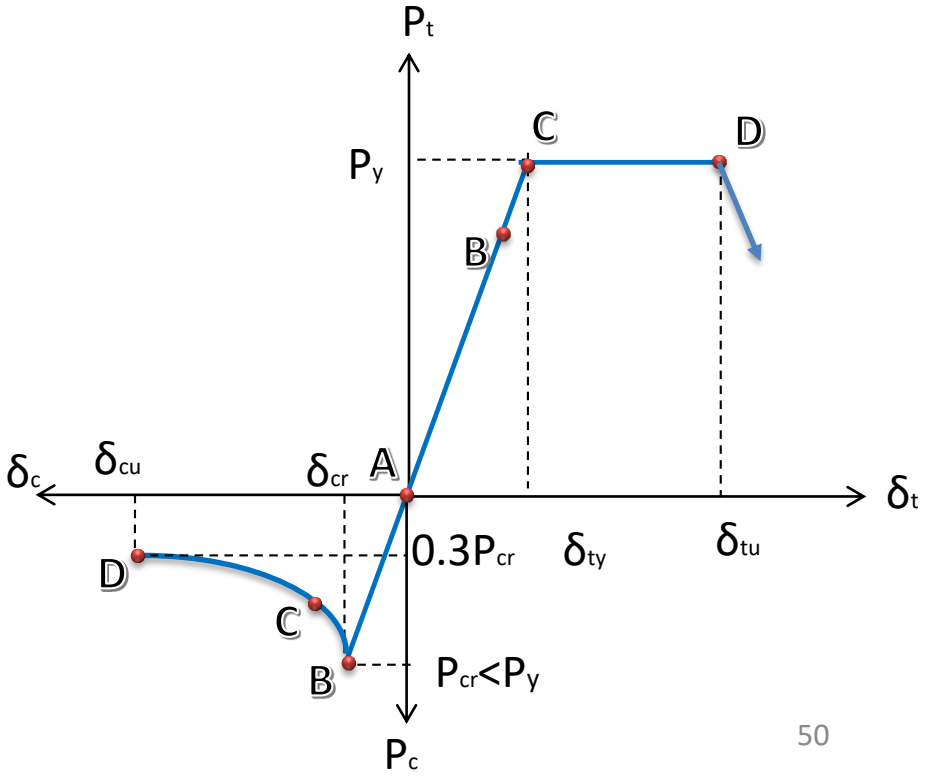
Representative capacity curve

A-B: Lateral movement is resisted by the total elastic stiffness, K (High stiffness, small drift)

B-C: Inelastic response with positive stiffness.

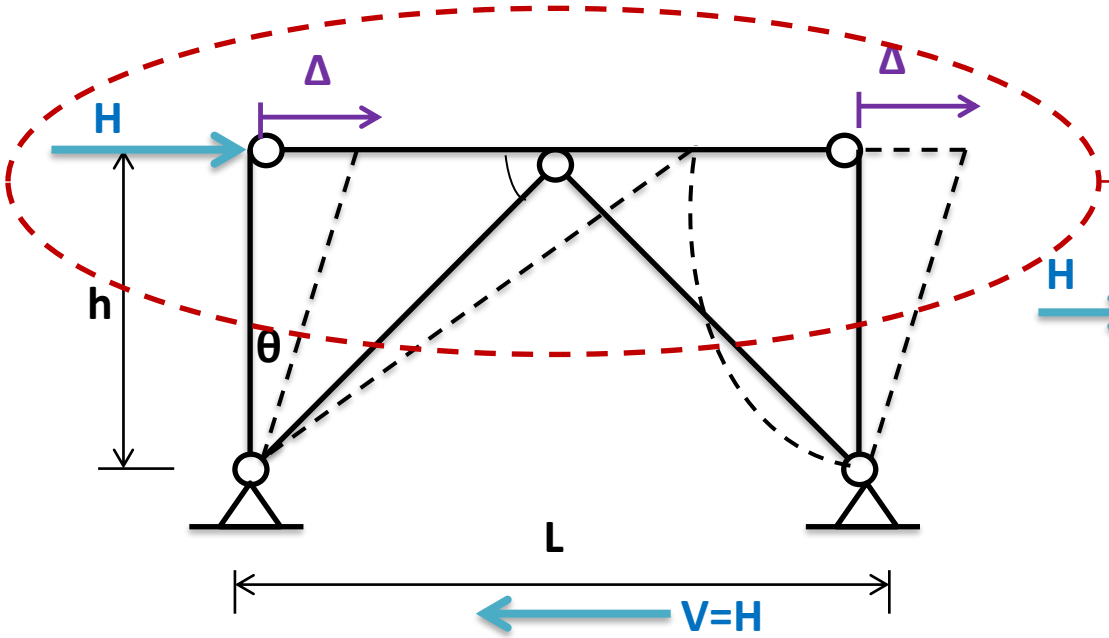
C-D: Inelastic response with negative stiffness until Δ_u when $\delta_t = \delta_{tu}$

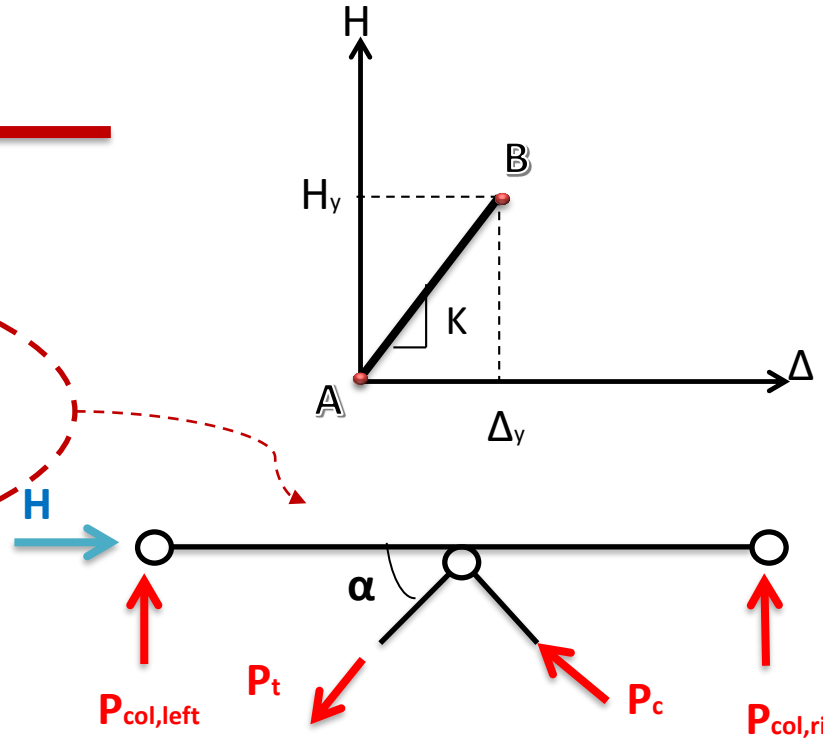




Inelastik Analiz

Axial vs. Lateral Strength Relationship





Elastic Stage
(Between A-B, $H \le H_y$)

Brace Forces

$$P_t = P_c$$

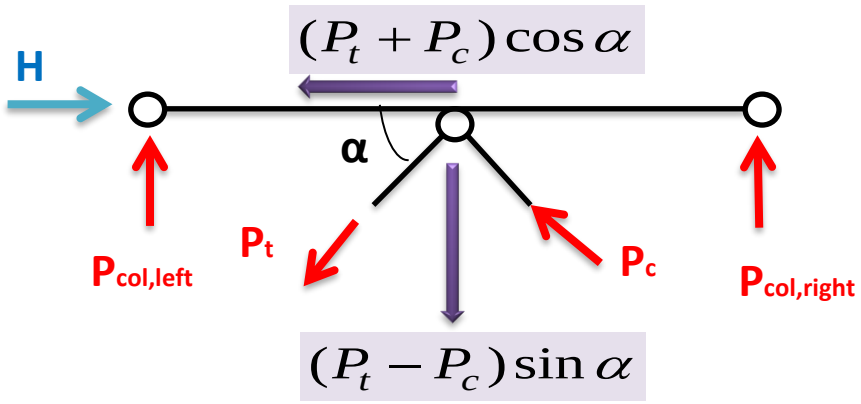
$$\sum F_x = 0 \Rightarrow H = (P_t + P_c)\cos\alpha$$

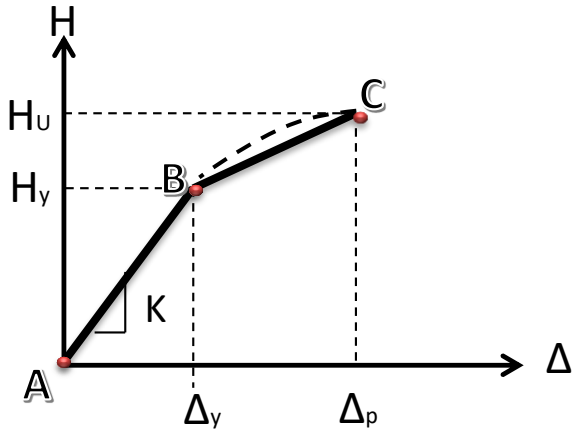
$$\sum F_y = 0 \Rightarrow P_{col,left} = P_{col,right} = 0$$

Beam: Axial force only (No bending)
Column: Zero force

Inelastik Analiz

Axial vs. Lateral Strength Relationship





Brace Forces

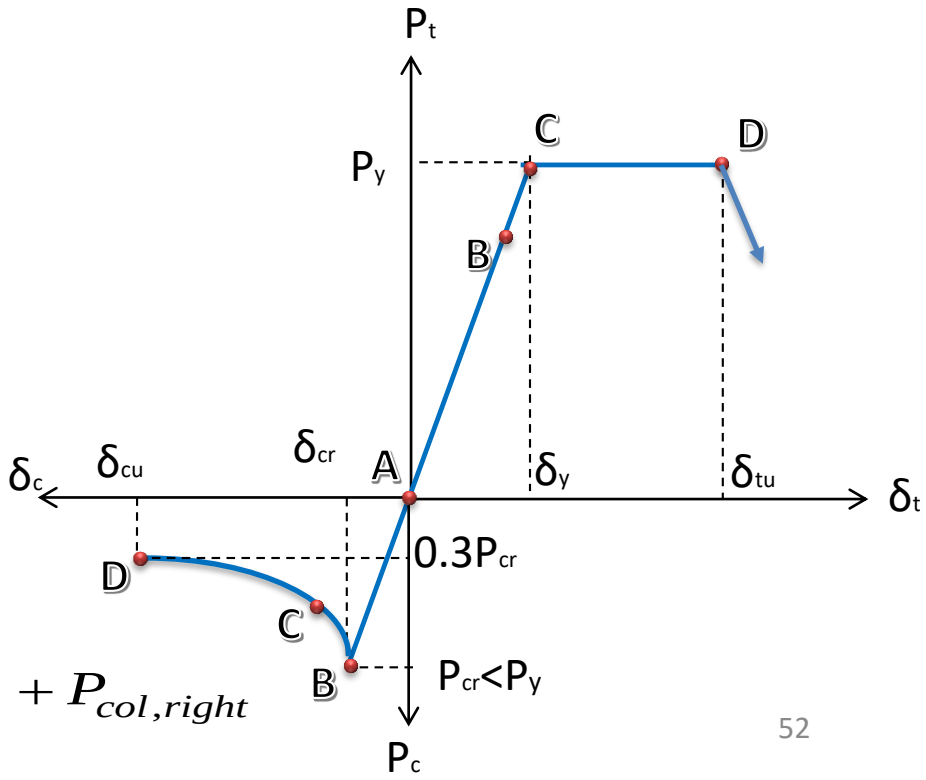
$$\left.\begin{aligned} P_t &= P_y = A_g F_y \\ P_c &< P_{cr} = A_g F_{cr} \end{aligned}\right\} P_t > P_c$$

Inelastic Stage (At Point C, H=H_u)

$$\sum F_x = 0 \Rightarrow H = (P_t + P_c)\cos\alpha$$

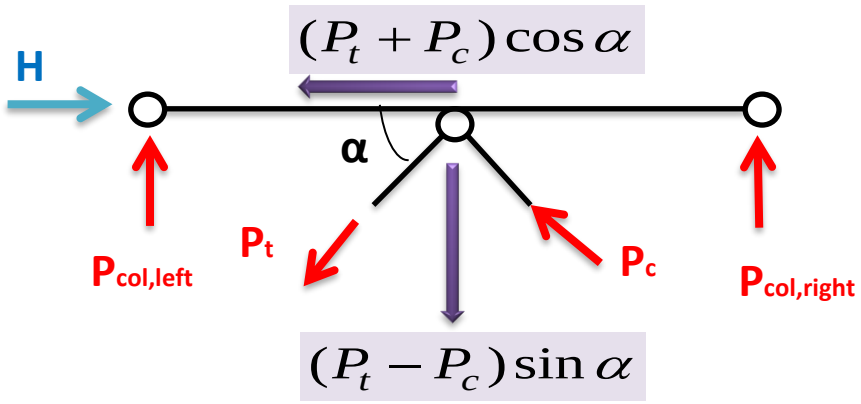
$$\sum M_0 = 0 \Rightarrow P_{col,left} = P_{col,right}$$

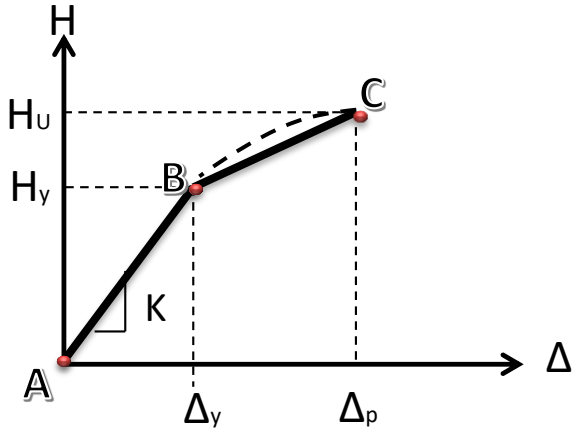
$$\sum F_y = 0 \Rightarrow (P_t - P_c)\sin\alpha = P_{col,left} + P_{col,right}$$



Inelastik Analiz

Axial vs. Lateral Strength Relationship

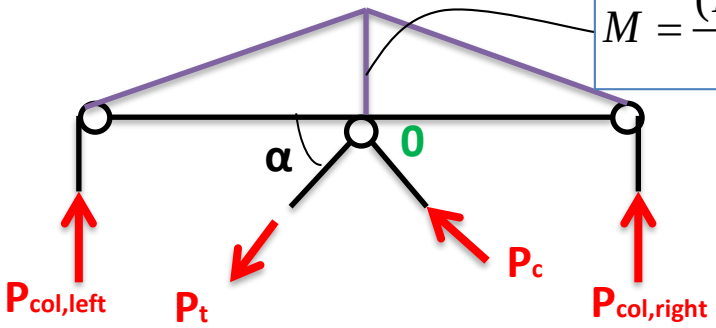




Brace Forces

$$\left.\begin{aligned} P_t &= P_y = A_g F_y \\ P_c &< P_{cr} = A_g F_{cr} \end{aligned}\right\} P_t > P_c$$

Beam&Column Response in Inelastic Stage (At Point C, H=H_u)



$$M = \frac{(P_t - P_c)\sin\alpha \times L}{4}$$

$$P_{col,right} = P_{col,left} = \frac{(P_t - P_c)\sin\alpha}{2}$$

Beam: Axial force+Bending moment

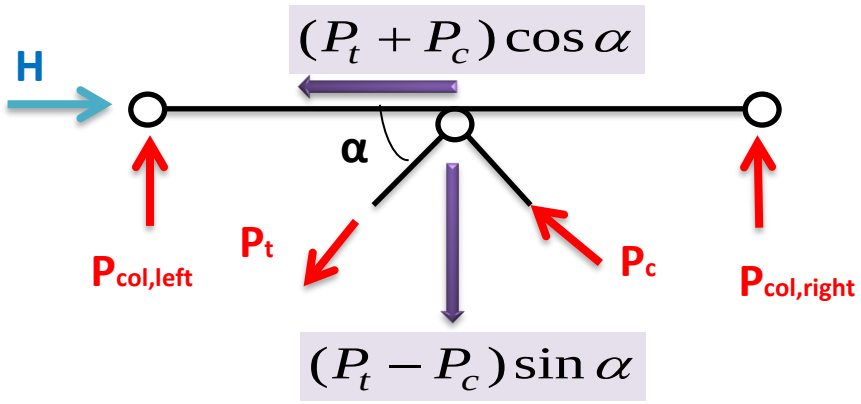
Column: Axial force only

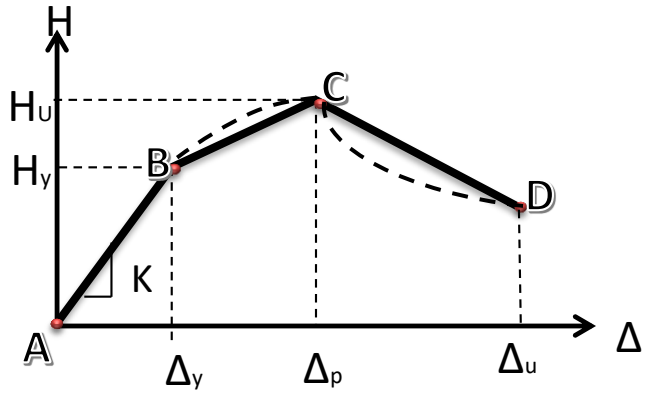
$$M_b = \frac{(A_g F_y - \lambda A_g F_{cr})\sin\alpha \times L}{4}$$

$$P_{col,right} = P_{col,left} = \frac{(A_g F_y - \lambda A_g F_{cr})\sin\alpha}{2}$$

Inelastik Analiz

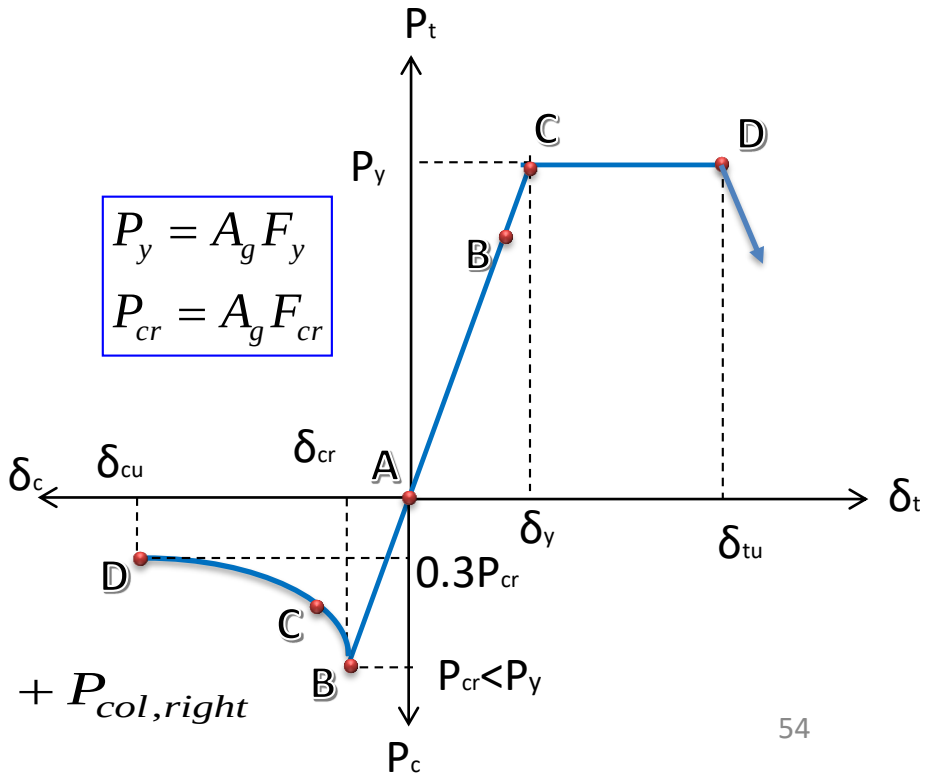
Axial vs. Lateral Strength Relationship





Brace Forces

$$\left.\begin{aligned} P_t &= P_y = A_g F_y \\ P_c &= 0.3P_{cr} = 0.3A_g F_{cr} \end{aligned}\right\} P_t > P_c$$



Final Stage (At Point D, $\Delta=\Delta_u$)

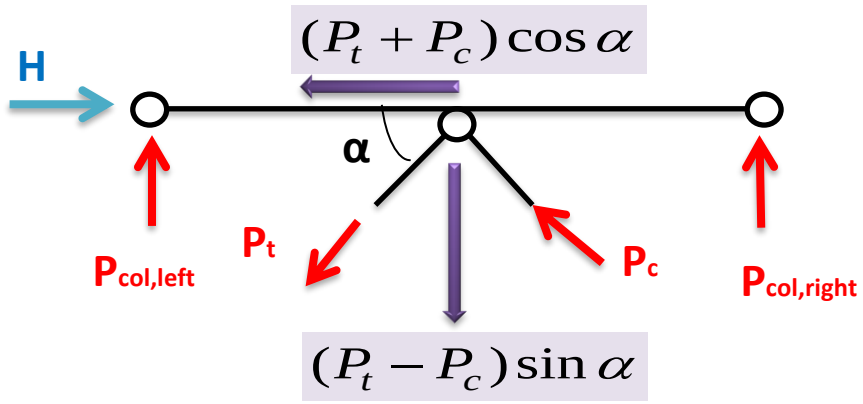
$$\sum F_x = 0 \Rightarrow H = (P_t + P_c)\cos\alpha$$

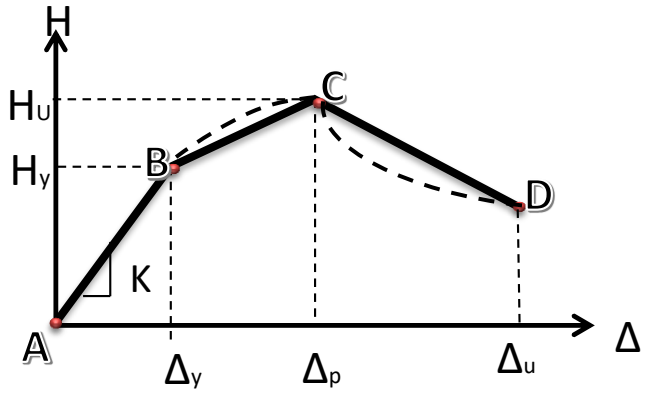
$$\sum M_0 = 0 \Rightarrow P_{col,left} = P_{col,right}$$

$$\sum F_y = 0 \Rightarrow (P_t - P_c)\sin\alpha = P_{col,left} + P_{col,right}$$

Inelastik Analiz

Axial vs. Lateral Strength Relationship

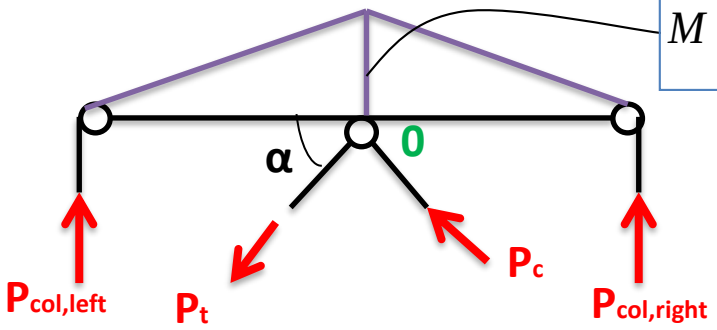




Brace Forces

$$\left.\begin{aligned} P_t &= P_y = A_g F_y \\ P_c &= 0.3P_{cr} = 0.3A_g F_{cr} \end{aligned}\right\} P_t > P_c$$

Beam&Column Response in Final Stage (At Point D, $\Delta=\Delta_u$)



$$M = \frac{(P_t - P_c)\sin\alpha \times L}{4}$$

$$P_{col,right} = P_{col,left} = \frac{(P_t - P_c)\sin\alpha}{2}$$

Beam: Axial force+Bending moment

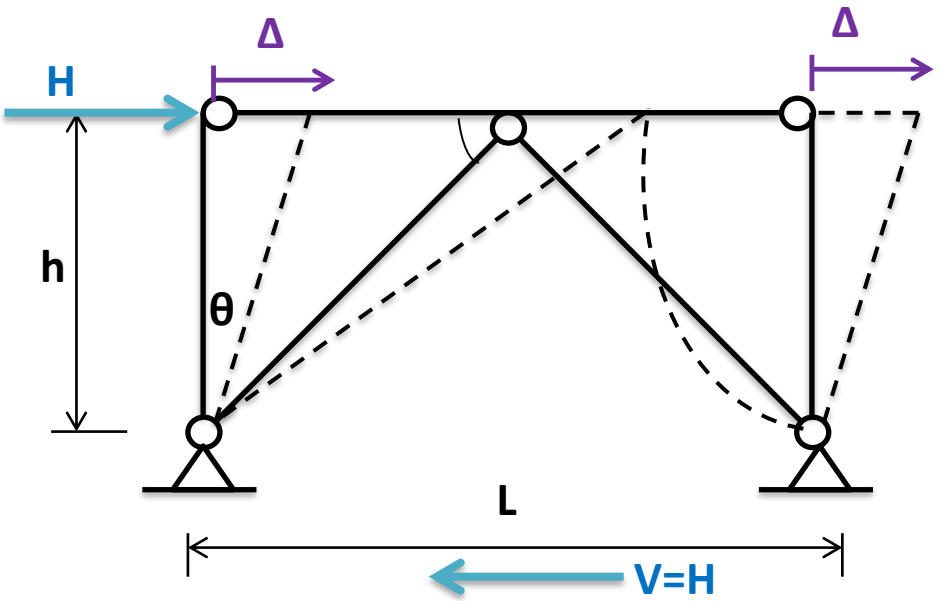
Column: Axial force only

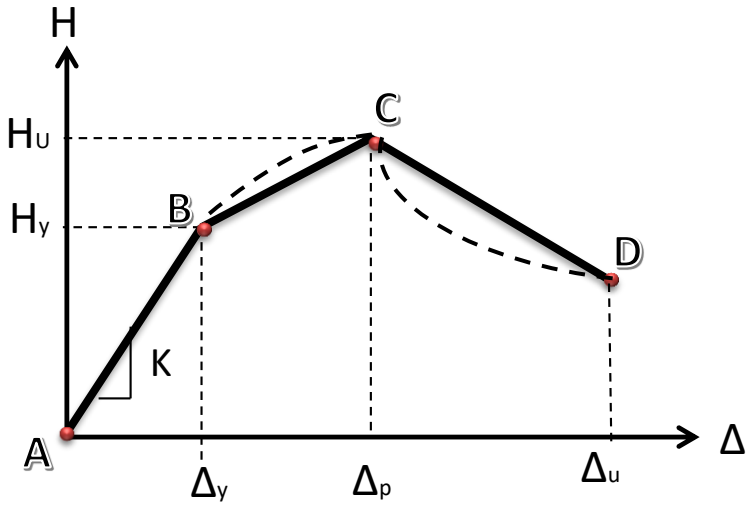
$$M_b = \frac{(A_g F_y - 0.3A_g F_{cr})\sin\alpha \times L}{4}$$

$$P_{col,right} = P_{col,left} = \frac{(A_g F_y - 0.3A_g F_{cr})\sin\alpha}{2}$$

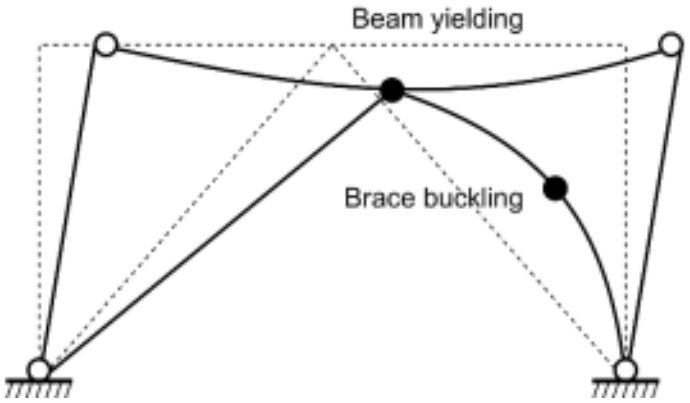
Inelastic Analysis

Discussion





- The energy dissipation mechanism in a CBF is post-buckling and post-yielding deformations of braces.
- To keep this "desirable" mechanism, the columns and beam have to be strong enough to remain elastic.
- The gusset plate needs to be much stronger than braces.



İstenmeyen Göçme Modları



Failed column base



Net section failure

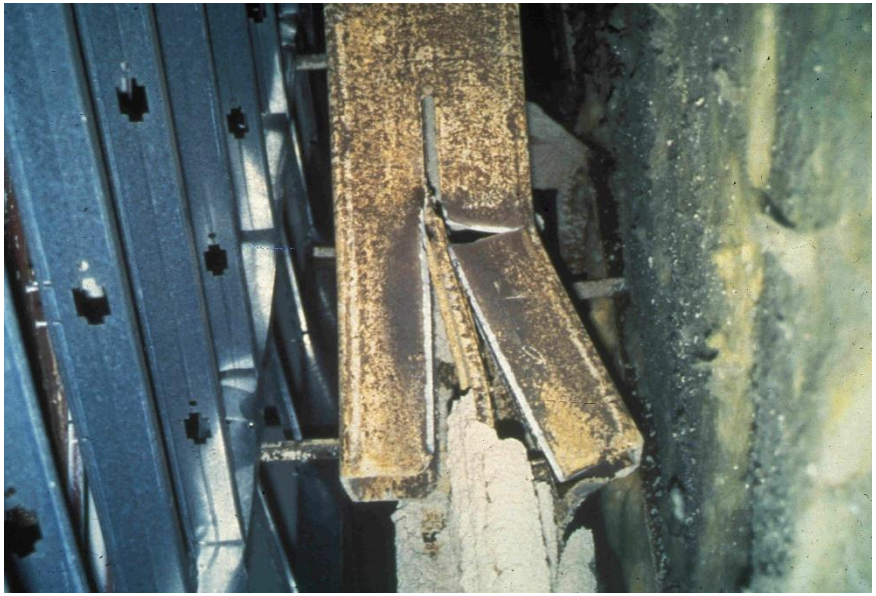


Gusset plate failure

İstenmeyen Göçme Modları



Column fracture



Connection failure

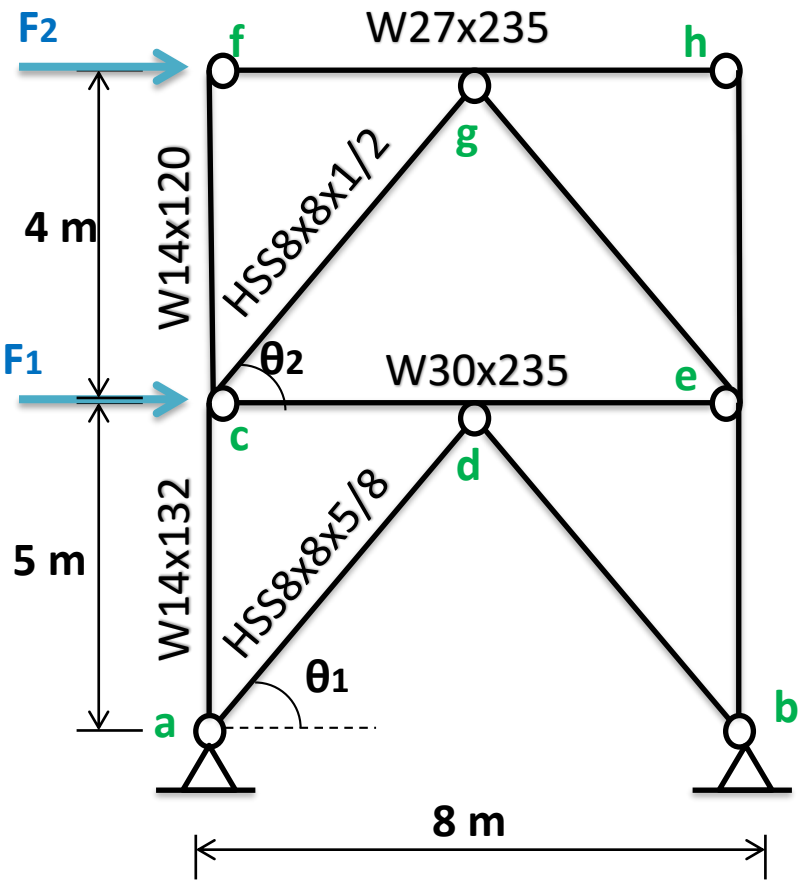


Connection failure



Connection failure

Statik İtme Analizi



İstenen:

Çerçeveyi mümkün olan en büyük yanal deplasmana kadar itin. Taban kesme kuvveti-çatı yer değiştirmesi ilişkisini çizin.

Verilenler:

- $2F_1=F_2$
- Her kattaki çaprazlar için aynı kesit kullanılmıştır.
- Kiriş ve kolon boyutları her katta değiştirilmiştir.
- Kiriş-kolon birleşimlerinin mafsallı olduğu varsayılmıştır.

W 30 x 235

Derinlik (in.) **Birim ağırlık (lb/ft)**

Statik İtme Analizi

Dört farklı göçme durumu göz önüne alınmıştır:

1) Çaprazların göçmesi
2) Kirişler, eksenel kuvvet ve eğilme momenti etkisinde kapasitesine ulaşabilir.
3) Kolonlar eksenel yük taşıma kapasitesine ulaşabilir
4) Birleşimlerde göçme (e.g., weld fracture or gusset plate fracture)

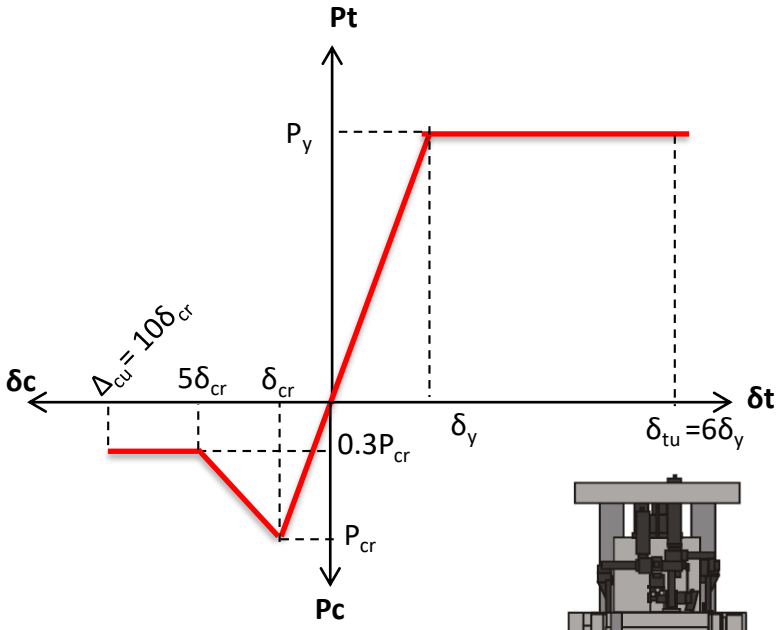
Kolon ve Kiriş Kapasiteleri (Deneysel)

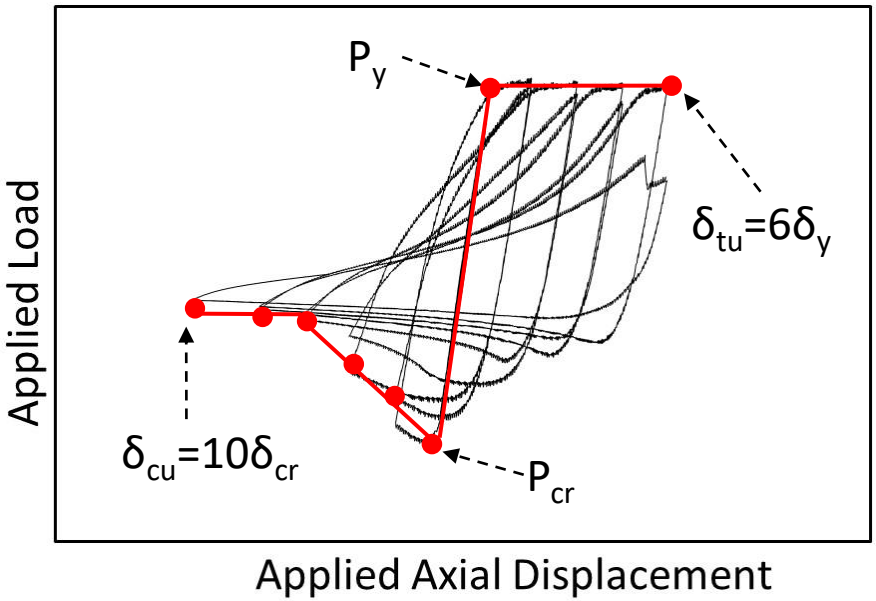
Kupon Testleriyle Elde Edilmiş Kapasiteler	1st Story Girder (W30x235)	2nd Story Girder (W27x235)	1st Story Column (W14x132)	2nd Story Column (W14x120)
Beklenen Eğilme Kapasitesi, M_{ce} (kN.m)	4935	4509	N/A	N/A
Beklenen Basınç Dayanımı, P_{cre} (kN)	14282	14314	7544	6880

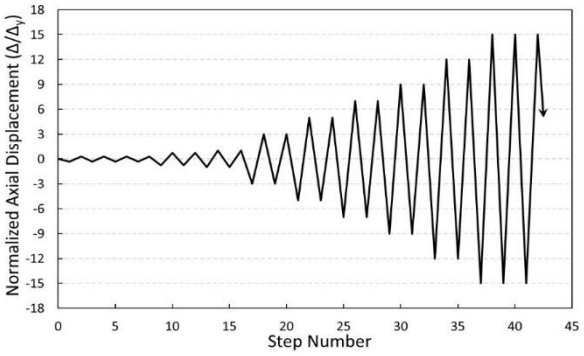
Statik İtme Analizi

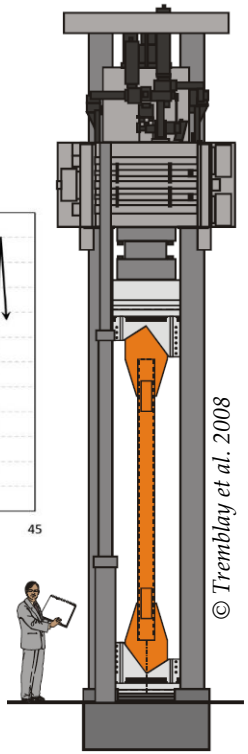
Deney Sonuçlarına Dayalı Çapraz Kapasiteleri

	HSS8x8x5/8	HSS8x8x1/2
Length	6.40 m	5.66 m
A_g	10,600 mm^2	8,710 mm^2
i	75.9 mm	77.2 mm
P_{cr}	1926 kN	1596 kN
P_y	2915 kN	2395 kN



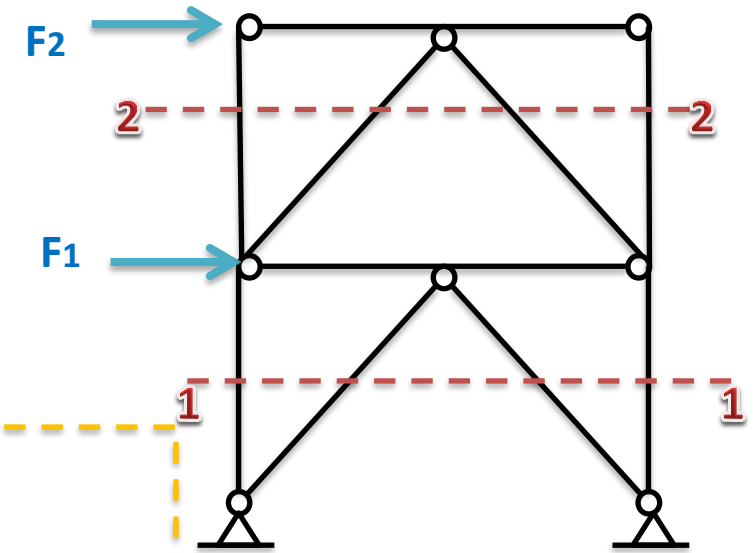


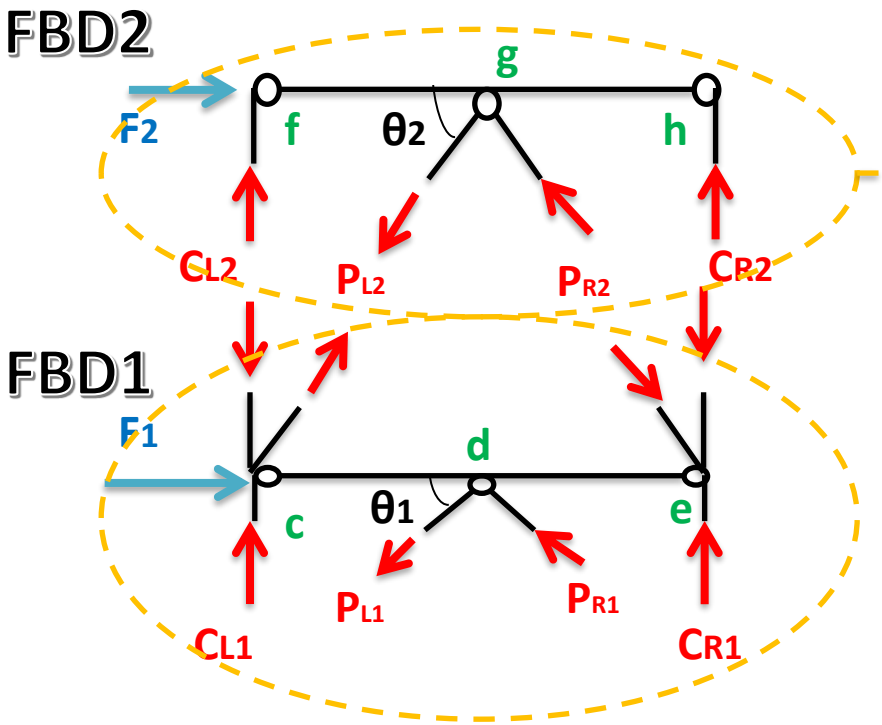




Serbest Cisim Diyagramları

Structural analysis to reveal some relationships between lateral load (H), drift ratio (Δ_i), and brace deformation (δ_i)





From FBD2

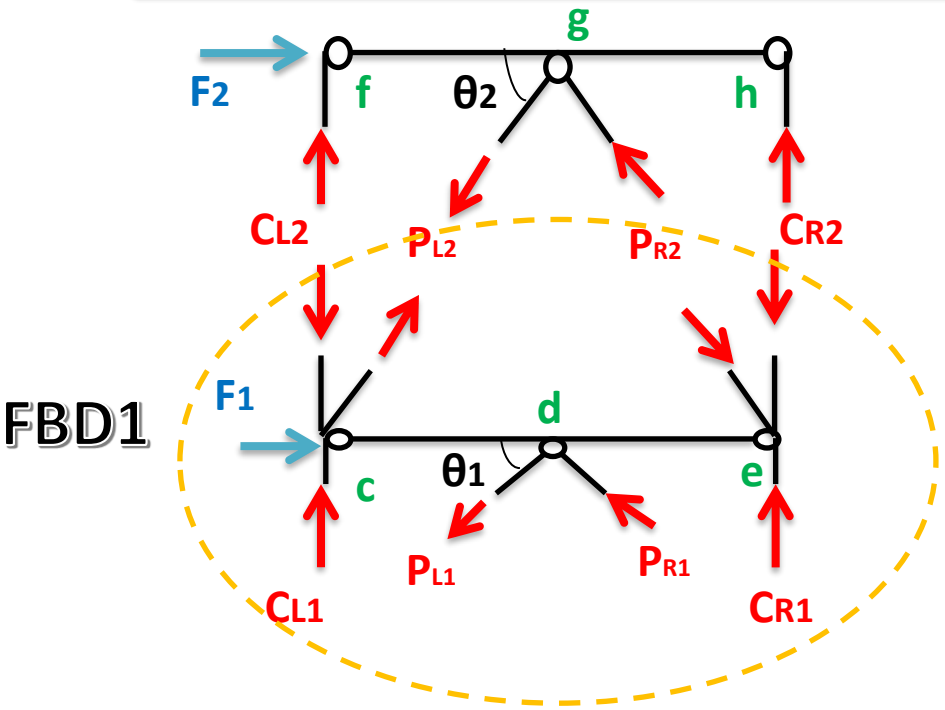
$$\sum F_x = 0 \rightarrow F_2 = (P_{L2} + P_{R2})\cos\theta_2$$

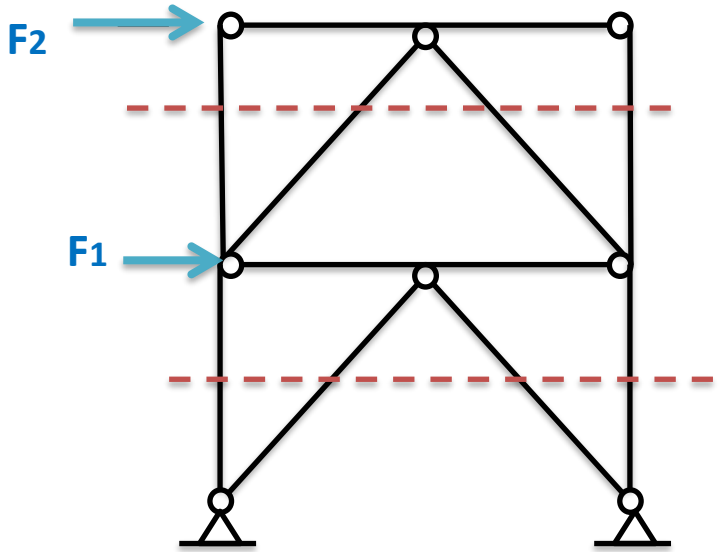
$$P_{R2} = P_{L2} = \frac{F_2}{2\cos\theta_2} \text{ (Elastic Stage)}$$

$$\sum M_g = 0 \rightarrow C_{R2} = C_{L2}$$

$$\sum F_y = 0 \rightarrow C_{R2} = C_{L2} = 0$$

Serbest Cisim Diyagramları





From FBD1

1

$$\sum F_x = 0 \rightarrow F_1 = (P_{L1} + P_{R1})\cos\theta_1 - (P_{L2} + P_{R2})\cos\theta_2$$

$$(P_{L1} + P_{R1})\cos\theta_1 = F_1 + F_2 \Rightarrow P_{L1} = P_{R1} = \frac{F_1 + F_2}{2\cos\theta_1}$$

2

$$\sum M_d = 0 \rightarrow C_{L1} \times \frac{L}{2} - C_{R1} \times \frac{L}{2} + P_{L2}\sin\theta_2 \frac{L}{2} + P_{R2}\sin\theta_2 \frac{L}{2} = 0$$

$$C_{L1} - C_{R1} = -2P_{L2}\sin\theta_2 = -2P_{R2}\sin\theta_2$$

3

$$\sum F_y = 0 \rightarrow C_{L1} + C_{R1} = 0$$

$$C_{L1} = -P_{L2}\sin\theta_2 \,(Tension)$$

$$C_{R1} = P_{L2}\sin\theta_2 \,(Compression)$$

Statik İtme Analizi

(1) Akma, burkulma ve çekme ve basınçtaki en büyük eksenel deformasyonlar

$$\delta_{cr} = \frac{P_{cr}}{k} = \frac{P_{cr}}{EA/L} = \frac{1926\times10^3}{\left[\left(2\times10^5\right)\times10,600/6400\right]} = 5.814^{mm}$$

$$\delta_{cu} = 10\delta_{cr} = 58.14^{mm}$$

$$\delta_y = \frac{P_y}{k} = \frac{P_y}{EA/L} = \frac{2915\times10^3}{\left[\left(2\times10^5\right)\times10,600/6400\right]} = 8.8^{mm}$$

$$\delta_{tu} = 6\delta_y = 52.8^{mm}$$

1. Kat Çaprazları HSS8x8x5/8

Statik İtme Analizi

(1) Akma, burkulma ve çekme ve basınçtaki en büyük eksenel deformasyonlar

$$\delta_{cr} = \frac{P_{cr}}{k} = \frac{P_{cr}}{EA/L} = \frac{1596\times10^3}{\left[\left(2\times10^5\right)\times 8,710/5660\right]} = 5.186^{mm}$$

$$\delta_{cu} = 10\delta_{cr} = 51.86^{mm}$$

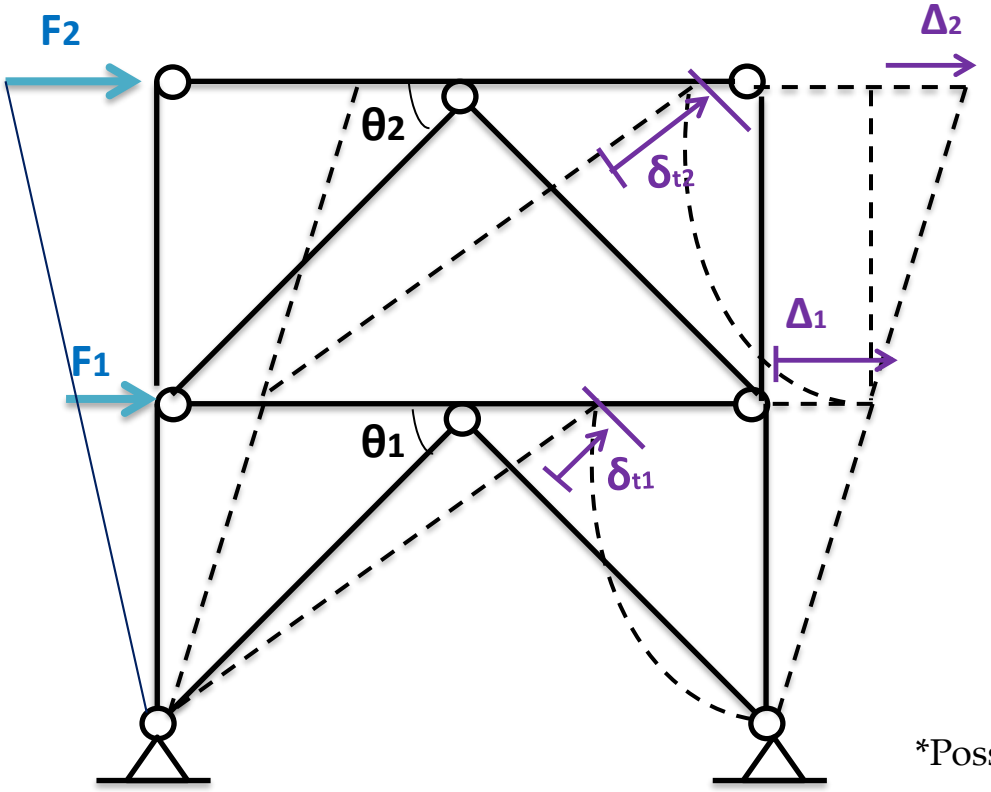
$$\delta_{y} = \frac{P_{y}}{k} = \frac{P_{y}}{EA/L} = \frac{2395\times10^3}{\left[\left(2\times10^5\right)\times 8,710/5660\right]} = 7.78^{mm}$$

$$\delta_{tu} = 6\delta_{y} = 46.7^{mm}$$

2. Kat Çaprazları HSS8x8x1/2

Statik İtme Analizi

Geometrik İlişkiler



$$\delta_{c1} = \delta_{t1} = \Delta_1 \cos\theta_1 \quad (1)$$

$$\delta_{c2} = \delta_{t2} = \Delta_2 \cos\theta_2 \quad (2)$$

$$U_{roof} = \Delta_1 + \Delta_2 \quad (3)$$

*Possible vertical deflections of the girders are neglected

Δ_1 : First story drift
Δ_2 : Second story drift
δ_{t1} : Axial deformation of first story brace in tension
δ_{t2} : Axial deformation of second story brace in tension
δ_{c1} : Axial deformation of first story brace in compression
δ_{c2} : Axial deformation of second story brace in compression

Statik İtme Analizi

(2) Akma, burkulma ve çekme ve basınçtaki en büyük eksenel deformasyonlara karşılık gelen göreli kat ötelemeleri

$$\Delta_{1,buckle} = \frac{\delta_{cr}}{\cos\theta_1} = \frac{5.814^{mm}}{\cos(51.3)} = 9.31^{mm}$$

$$\Delta_{1,cu} = \frac{\delta_{cu}}{\cos\theta_1} = \frac{58.14^{mm}}{\cos(51.3)} = 93.1^{mm}$$

$$\Delta_{1,yield} = \frac{\delta_{y}}{\cos\theta_1} = \frac{8.8^{mm}}{\cos(51.3)} = 14.07^{mm}$$

$$\Delta_{1,\max} = \Delta_{1,tu} = \frac{\delta_{tu}}{\cos\theta_1} = \frac{52.8^{mm}}{\cos(51.3)} = 84.5^{mm}$$

$$\delta_{c1} = \delta_{t1} = \Delta_1 \cos\theta_1$$

1. Kat Çaprazları HSS8x8x5/8

Statik İtme Analizi

(2) Akma, burkulma ve çekme ve basınçtaki en büyük eksenel deformasyonlara karşılık gelen göreli kat ötelemeleri

$$\Delta_{2,buckle} = \frac{\delta_{cr}}{\cos\theta_2} = \frac{5.186^{mm}}{\cos(45)} = 7.33^{mm}$$

$$\Delta_{2,cu} = \frac{\delta_{cu}}{\cos\theta_2} = \frac{51.86^{mm}}{\cos(45)} = 73.3^{mm}$$

$$\Delta_{2,yield} = \frac{\delta_y}{\cos\theta_2} = \frac{7.78^{mm}}{\cos(45)} = 11^{mm}$$

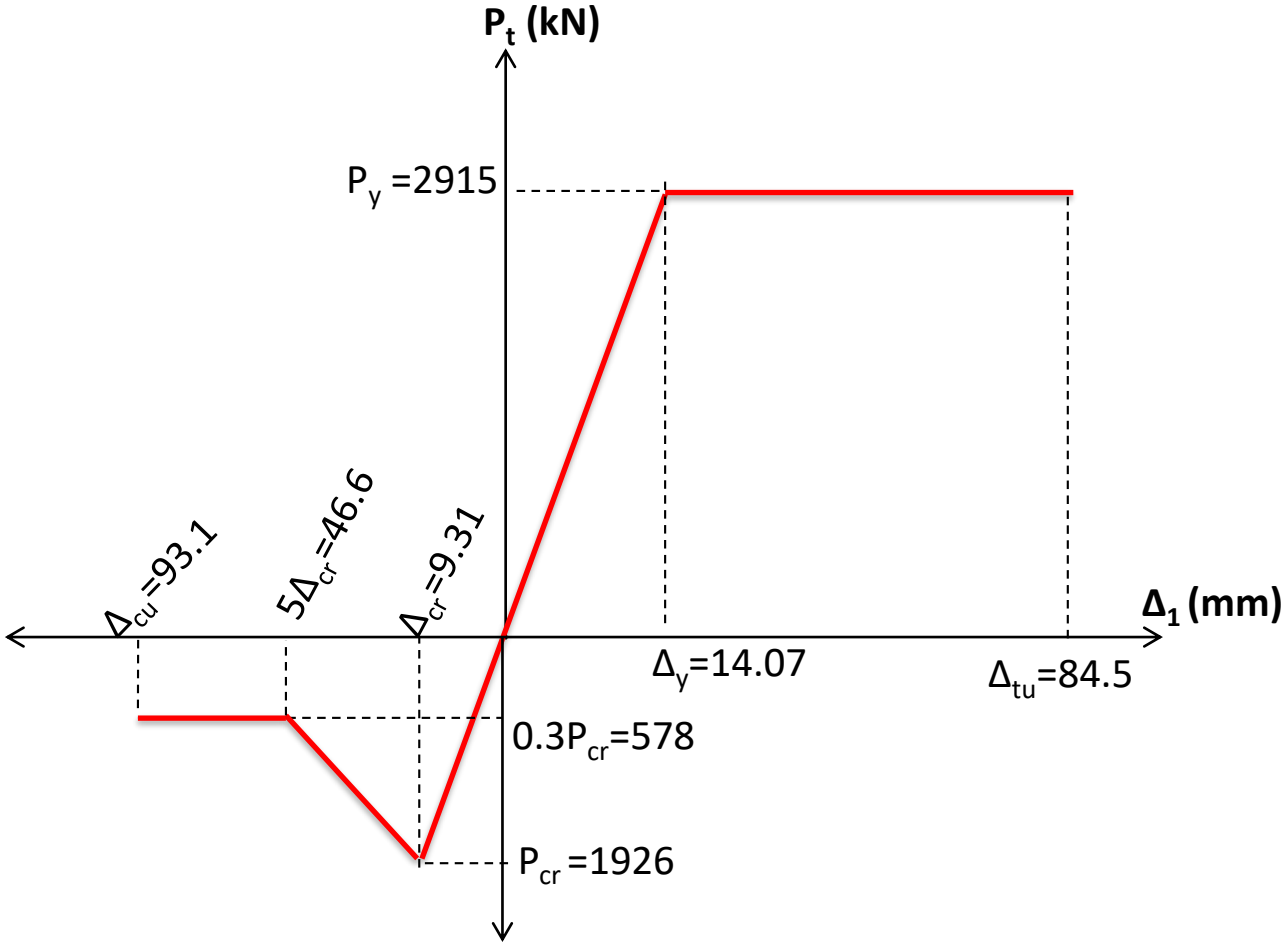
$$\Delta_{2,\max} = \Delta_{2,tu} = \frac{\delta_{tu}}{\cos\theta_2} = \frac{52.8^{mm}}{\cos(45)} = 66^{mm}$$

$$\delta_{c2} = \delta_{t2} = \Delta_2 \cos\theta_2$$

2. Kat Çaprazları HSS8x8x1/2

Statik İtme Analizi

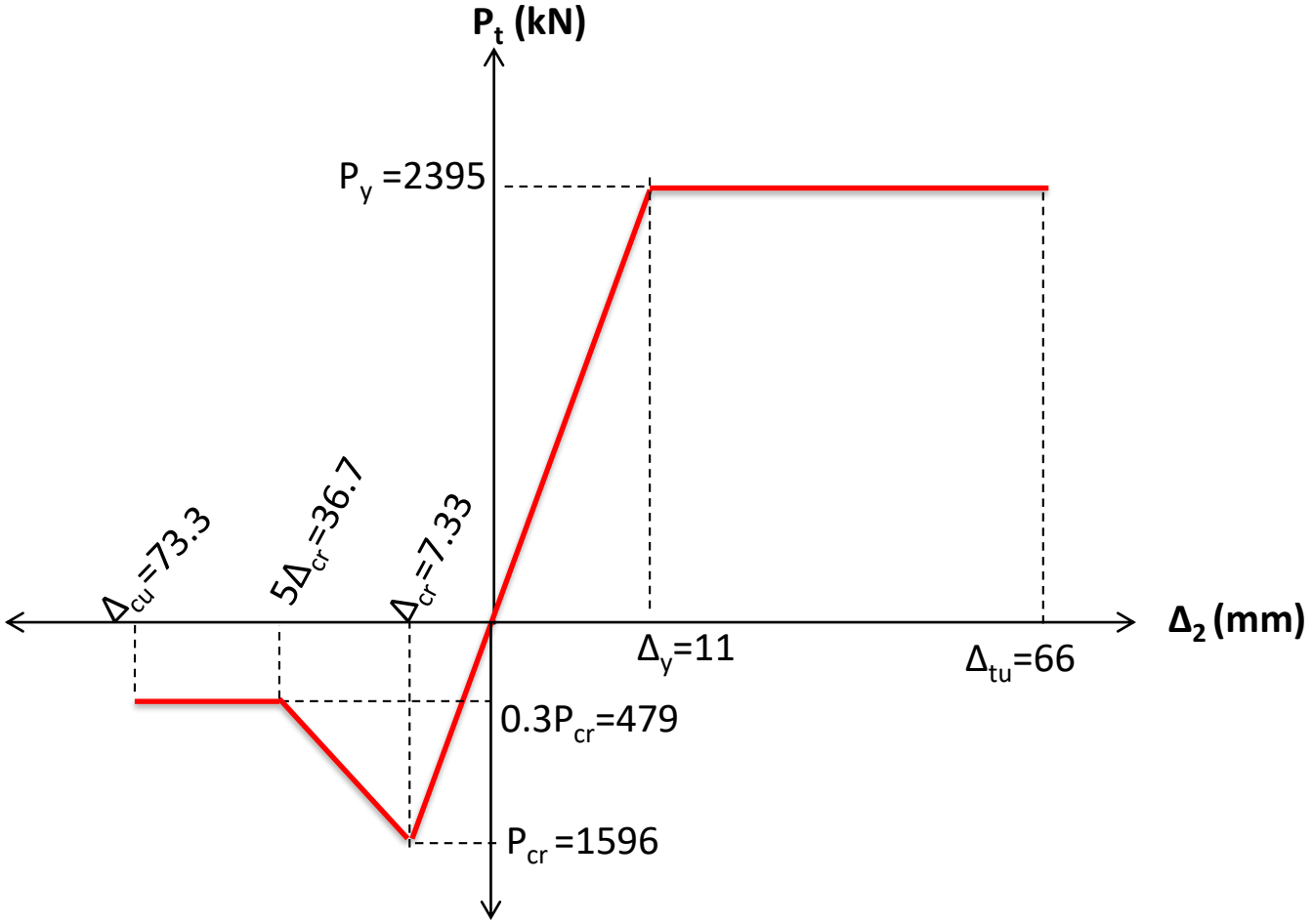
(3) İdealize edilmiş 1. kat çapraz davranışı (Göreli kat ötelemesi-Eksenel Dayanım)



1. Kat Çaprazları: HSS8x8x5/8

Statik İtme Analizi

(3) İdealize edilmiş 2. kat çapraz davranışı (Göreli kat ötelemesi-Eksenel Dayanım)



2. Kat Çaprazları: HSS8x8x1/2

Statik İtme Analizi

(4) Adım Adım Analiz

Step 1: Çaprazların Burkulması

	1st Story HSS8x8x5/8	2nd Story HSS8x8x1/2
P_{cr}	1926 kN	1596 kN
P_y	2915 kN	2395 kN

İkinci Kat

Brace Forces: P_{R2} and P_{L2}

$$P_{R2} = P_{L2} = \frac{F_2}{2\cos\theta_2} = \frac{2F}{2\cos 45}$$

When second story brace buckles

$$F = P_{cr2}\cos 45 = 1596 \times \cos 45$$

$$F = 1128.5^{kN}$$

Birinci Kat

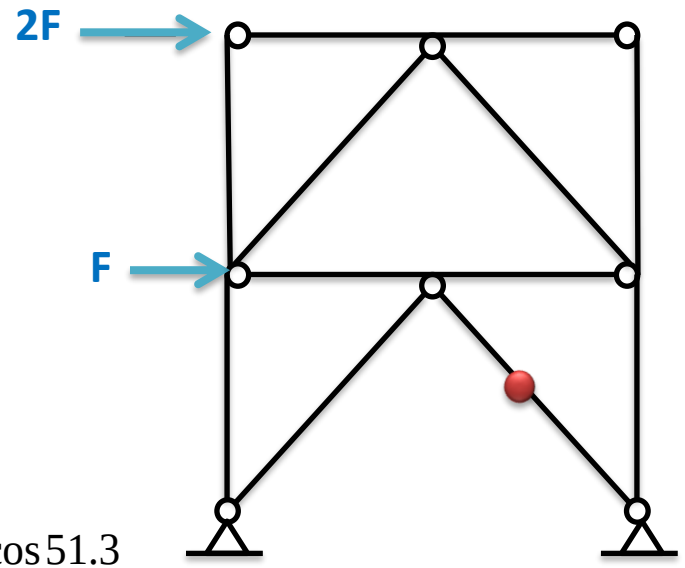
Brace Forces: P_{R1} and P_{L1}

$$P_{R1} = P_{L1} = \frac{F_1 + F_2}{2\cos\theta_1} = \frac{3F}{2\cos 51.3}$$

When first story brace buckles

$$F = 2/3 P_{cr1}\cos 51.3 = 2/3 \times 1926 \times \cos 51.3$$

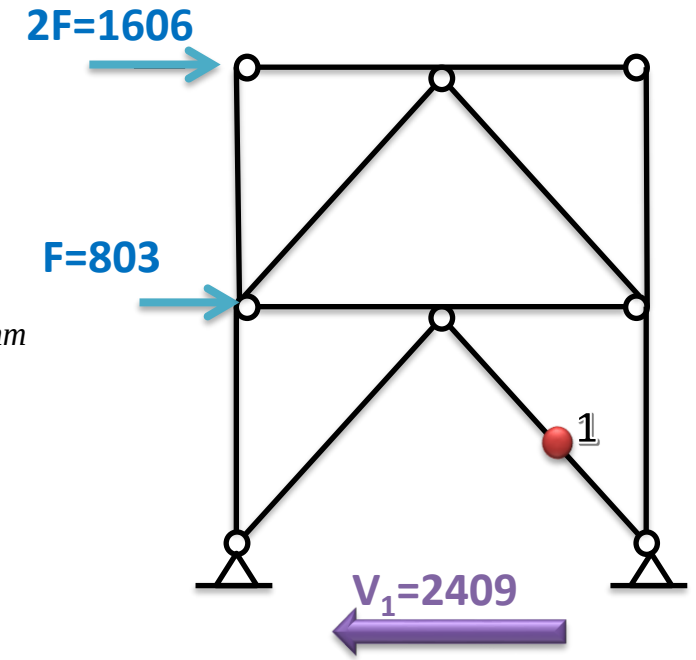
$$F = 803^{kN}$$



Birinci Kat Çaprazı Burkulur

Statik İtme Analizi

(4) Adım Adım Analiz

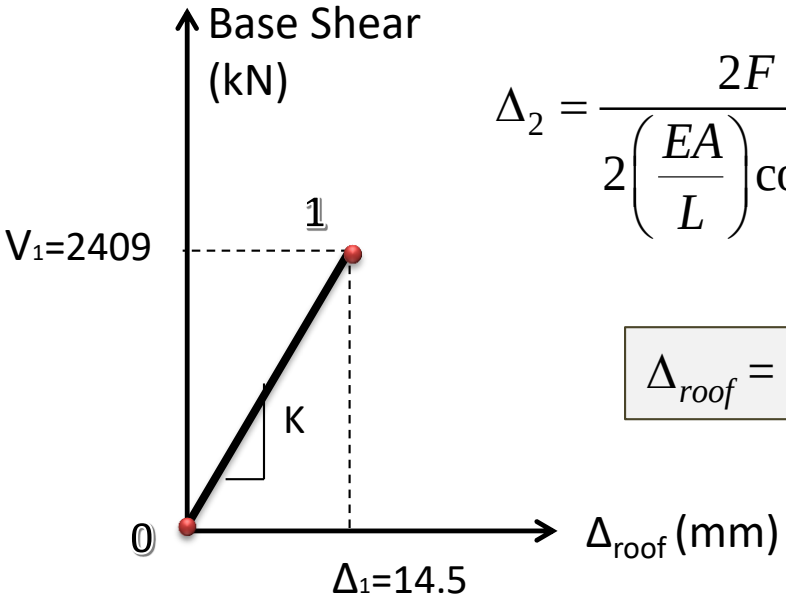


Step 1: Çaprazın Burkulması

Birinci kat çaprazı burkulduğu anda (1 Noktası)

First Story Drift, $\Delta_{1,buckle} = \dfrac{\delta_{cr1}}{\cos\theta_1} = \dfrac{5.814^{mm}}{\cos(51.3)} = 9.3^{mm}$

Second Story Drift, $\Delta_2 = \dfrac{2F}{K_{story,1}} = \dfrac{2F}{2k_{brace}\cos^2\theta_2}$



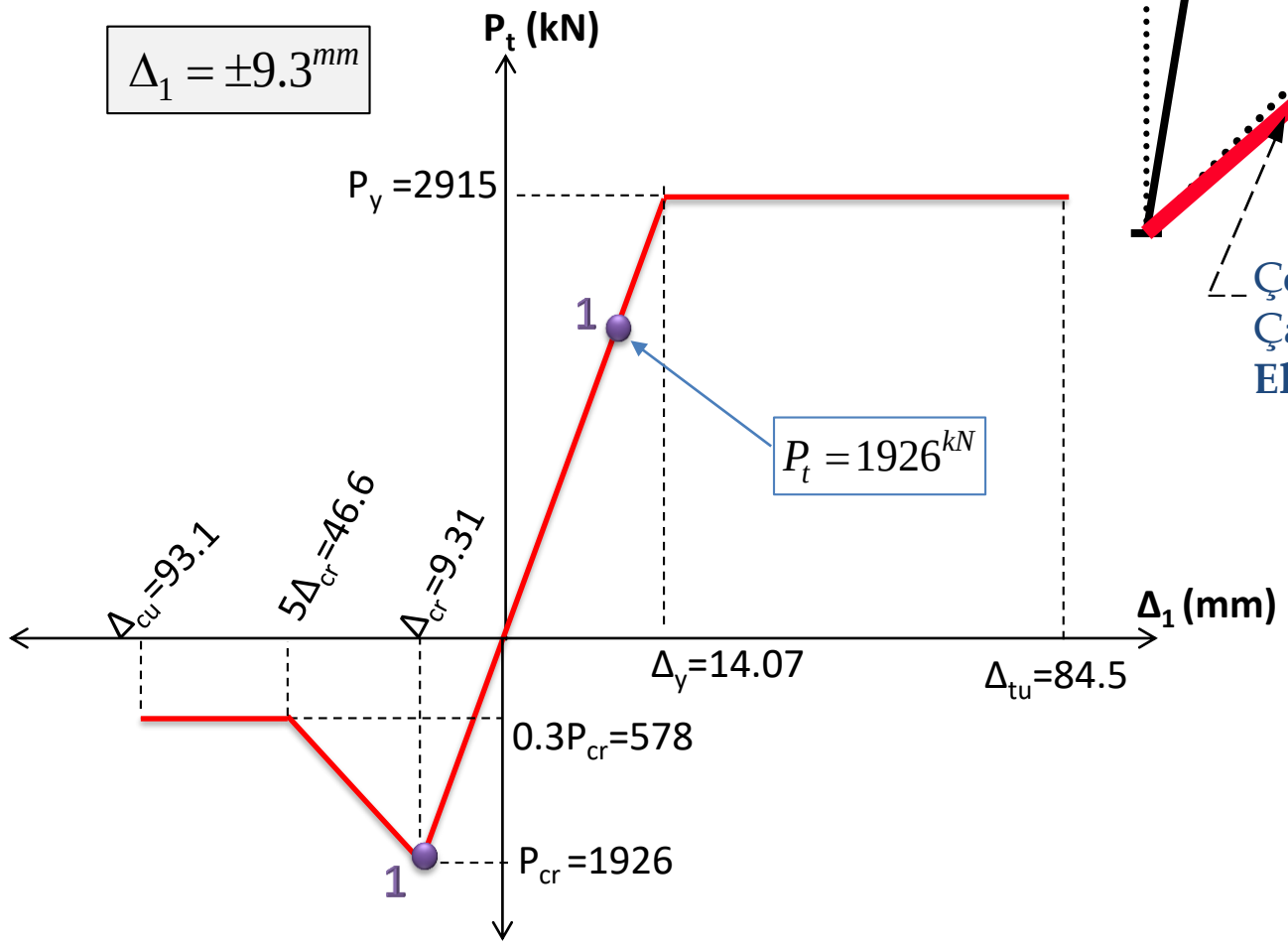
$$\Delta_2 = \frac{2F}{2\left(\frac{EA}{L}\right)\cos^2\theta_2} = \frac{2\times 803\times 10^3}{2\left(\frac{\left(2\times 10^5\right)\times 8,710}{5660}\right)\cos^2 45} = 5.2^{mm}$$

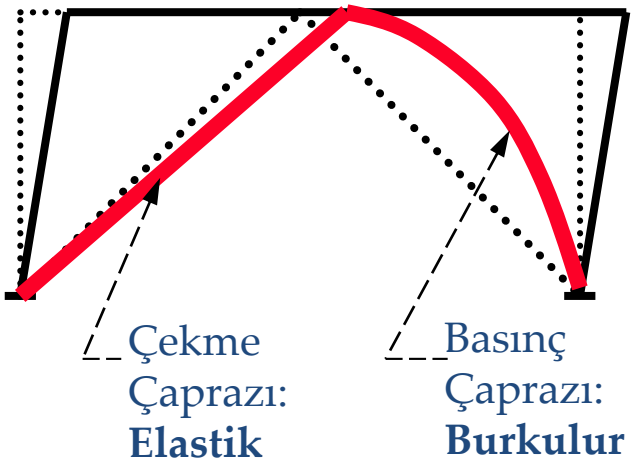
$$\Delta_{roof} = \Delta_1 + \Delta_2 = 9.3 + 5.2 = 14.5^{mm}$$

$$V_1 = 2409^{kN}$$

Statik İtme Analizi

1 Noktasında (1. Kat Çaprazı Burkulur)

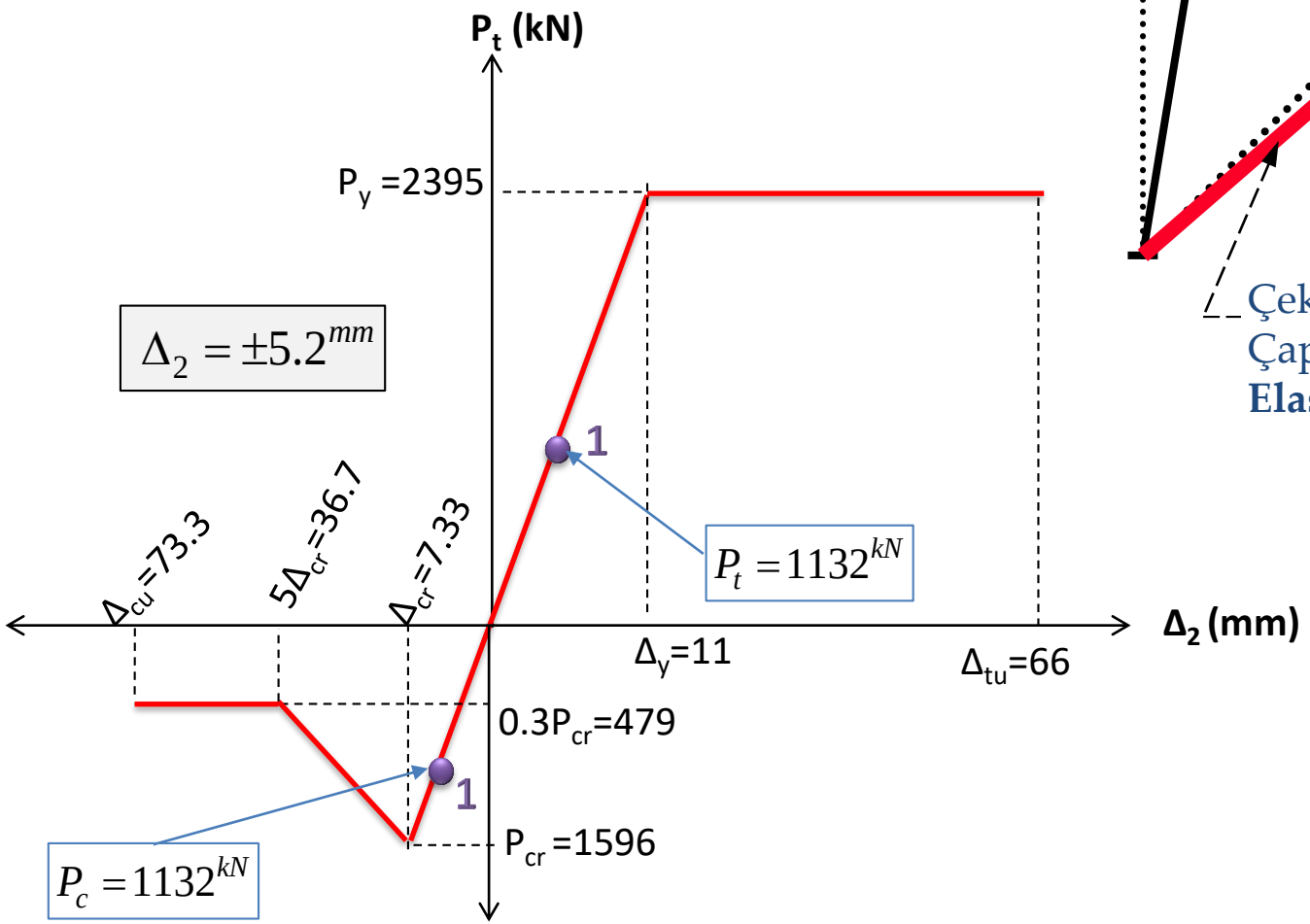


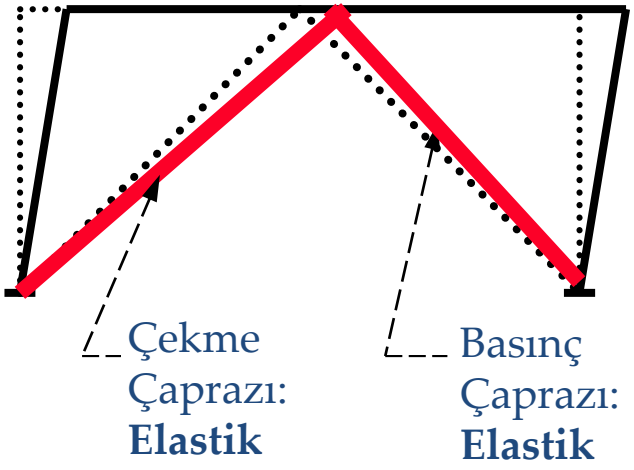


1st Story Brace: HSS8x8x5/8

Statik İtme Analizi

1 Noktasında (1. Kat Çaprazı Burkulur)





2nd Story Brace: HSS8x8x1/2

Statik İtme Analizi

Step 2: 2. kat çaprazı burkulabilir veya 1. kat çaprazı akabilir

İkinci Kat Çaprazı Burkulur

Brace Forces: P_{R2} and P_{L2}

$$P_{R2} = P_{L2} = \frac{2\Delta F}{2\cos 45}$$

When the second story brace buckles

$$\Delta F = P_{residual\,2}\cos 45 = \left(1596 \times \frac{7.3-5.2}{7.3}\right) \times \cos 45$$

$$\Delta F = 325^{kN}$$

Birinci Kat Çaprazı Akar

Brace Forces: P_{R1} (Comp.) and P_{L1} (Tension)

When $\Delta_1 = \Delta_{y1} = 14.07^{mm}$

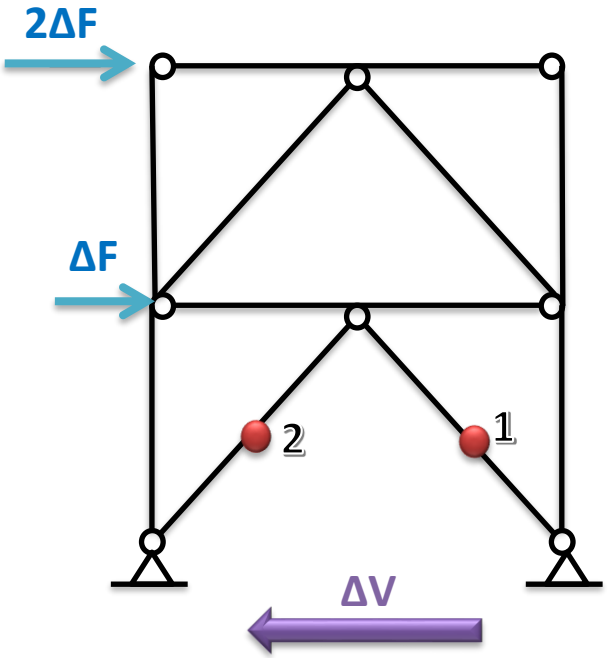
$$P_{R1} \neq P_{L1} \Rightarrow \Delta P_{R1} + \Delta P_{L1} = \frac{3\Delta F}{\cos 51.3}$$

$$P_{L1} = P_y = 2915^{kN} \rightarrow \Delta P_{L1} = 2915 - 1926 = 989kN$$

$$P_{R1} < P_{cr1} \rightarrow \Delta P_{R1} = -\frac{(1926-578)}{(46.6-9.3)} \times (14.07-9.3) = -172.4^{kN}$$

$$\Delta F = 170^{kN}$$

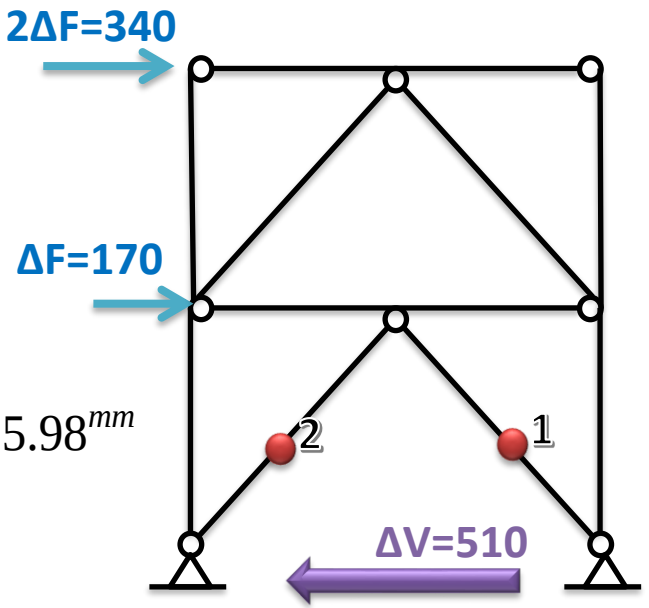
$$\Delta P_{R1} + \Delta P_{L1} = 989^{kN} - 172.4^{kN} = \frac{3\Delta F}{\cos 51.3} \rightarrow \Delta F = 170^{kN}$$



1. Kat Çaprazı Akar

Statik İtme Analizi

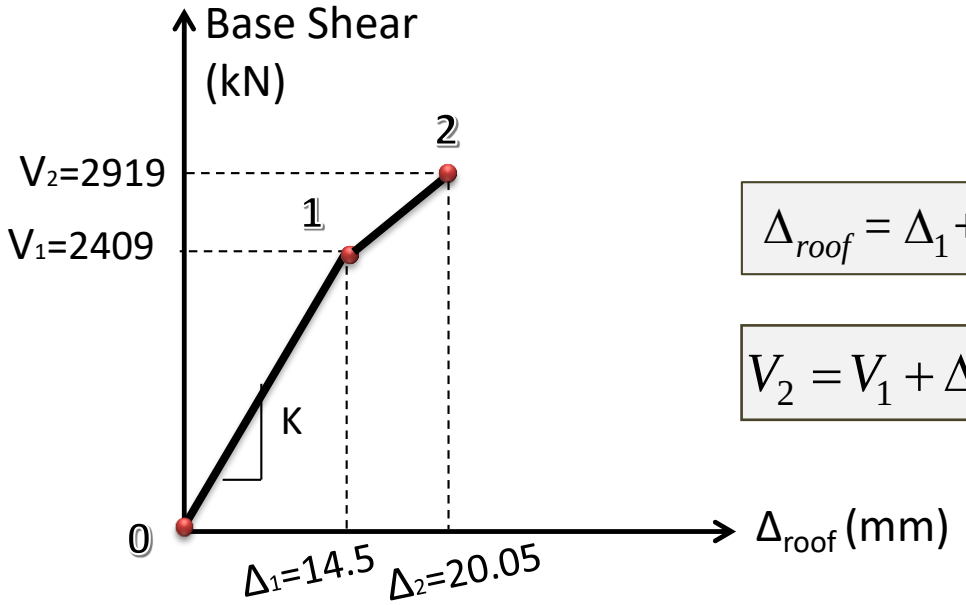
Step 2: Birinci Kat Çaprazı Akar (2. Nokta)



First Story Drift, $\Delta_1 = \Delta_{1,yield} = 14.07^{mm}$

Second Story Drift, $\Delta_2 = 5.2 + \dfrac{2\Delta F}{2K_{elastic}}$

$$\Delta_2 = 5.2 + \frac{340}{2\times\left(2395^{kN}/11^{mm}\right)} = 5.2^{mm} + 0.78^{mm} = 5.98^{mm}$$

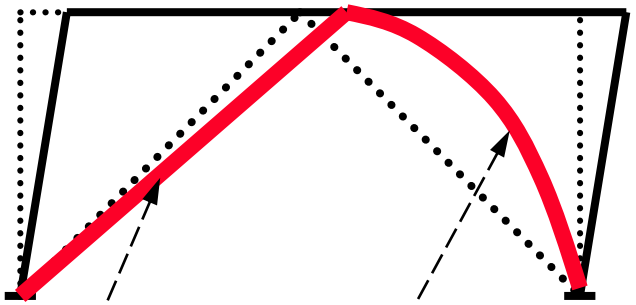


$$\Delta_{roof} = \Delta_1 + \Delta_2 = 14.07 + 5.98 = 20.05^{mm}$$

$$V_2 = V_1 + \Delta V = 2409 + 510 = 2919^{kN}$$

Statik İtme Analizi

2 Noktasında (Birinci Kat Çaprazı Akar)



Çekme Çaprazı: **Akma**

Baısnç Çaprazı: **Burkuldu**

$\Delta_1 = \pm 14.07^{mm}$

$K_{postyield} = 0$

$K_{elastic} = \frac{2915}{14.07} = 207.18^{kN/mm}$

$P_c^2 = 1754^{kN}$

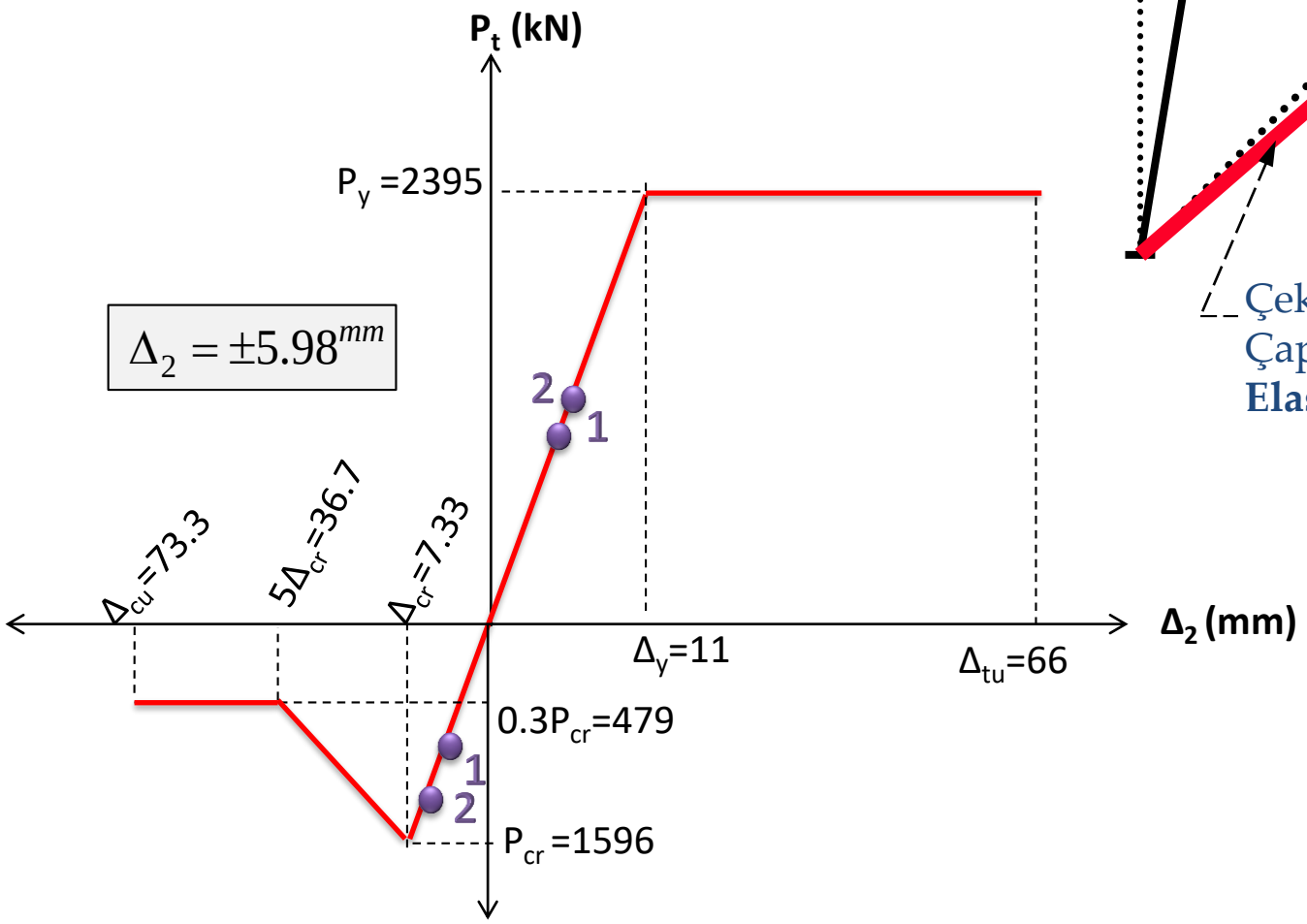
$K_{postbuckling} = -\frac{1926-578}{46.6-9.31} = -36.14^{kN/mm}$

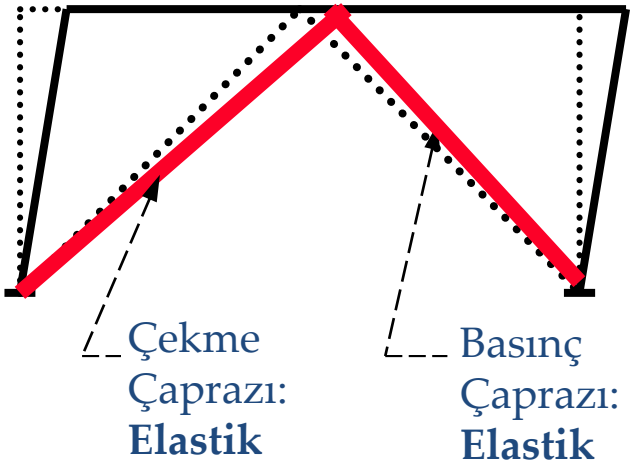
P_t (kN), Δ_1 (mm), P_y=2915, P_{cr}=1926, $0.3P_{cr}$=578, Δ_{cu}=93.1, $5\Delta_{cr}$=46.6, Δ_{cr}=9.31, Δ_y=14.07, Δ_{tu}=84.5

1st Story Brace: HSS8x8x5/8

Statik İtme Analizi

2 Noktasında (Birinci Kat Çaprazı Akar)





2nd Story Brace: HSS8x8x1/2

Statik İtme Analizi

Step 3: 2. Kat Çaprazı Burkulabilir veya Birinci Kat Basınç Çaprazı $5\Delta_{cr}=46.6^{mm}$ ulaşabilir

İkinci Kat Çaprazının Burkulması için

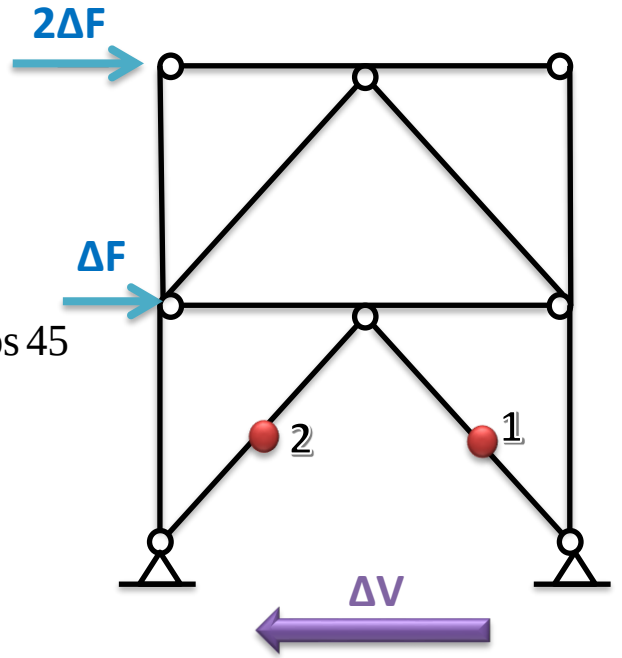
Brace Forces: P_{R2} and P_{L2}

$$P_{R2} = P_{L2} = \frac{2\Delta F}{2\cos 45}$$

When the second story brace buckles

$$\Delta F = P_{residual3}\cos 45 = \left(1596 \times \frac{7.3-5.98}{7.3}\right) \times \cos 45$$

$$\Delta F = 204^{kN}$$



1. Kat Basınç Çaprazı $5\Delta_{cr}$ ulaşır

Birinci Kat Çaprazının $5\Delta_{cr}=46.6^{mm}$ Ulaşması için

Brace Forces: P_{R1} (Comp.) and P_{L1} (Tension)

When $\Delta_1 = 5\Delta_{cr1} = 46.6^{mm}$

$$P_{R1} \neq P_{L1} \Rightarrow \Delta P_{R1} + \Delta P_{L1} = \frac{3\Delta F}{\cos 51.3}$$

$$P_{L1} = P_y = 2915^{kN} \rightarrow \Delta P_{L1} = 0$$

$$P_{R1} < P_{cr1} \rightarrow \Delta P_{R1} = -(1754-578) = -1176^{kN}$$

$$\Delta P_{R1} + \Delta P_{L1} = 0 - 1176^{kN} = \frac{3\Delta F}{\cos 51.3} \rightarrow \Delta F = -245^{kN}$$

$\Delta F = -245^{kN}$

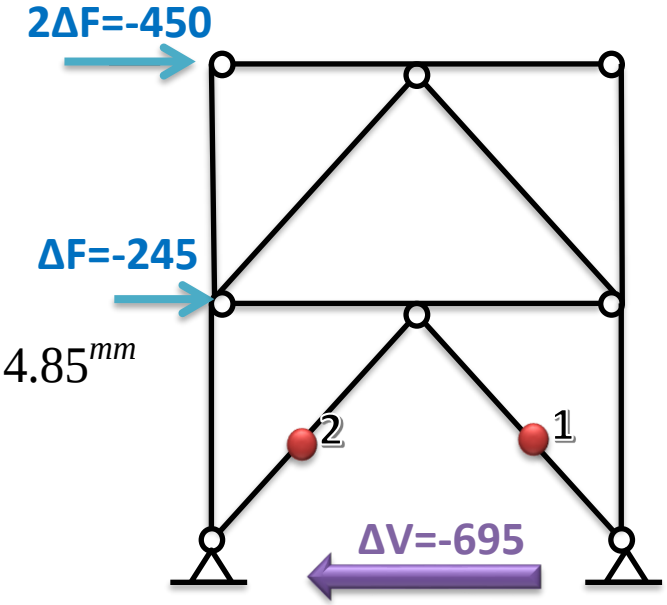
Statik İtme Analizi

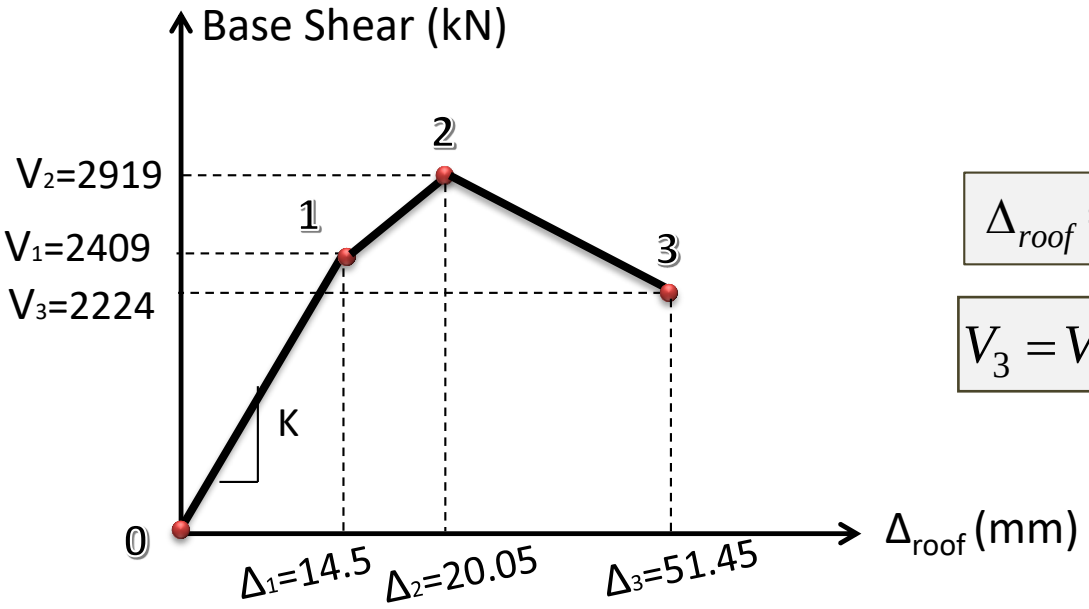
Step 3: 1. Kat Basınç Çaprazı $5\Delta_{cr}$ ulaştığında (3 Noktası)

First Story Drift, $\Delta_1 = 5\Delta_{1,cr} = 46.6^{mm}$

Second Story Drift, $\Delta_2 = 5.98 + \dfrac{2\Delta F}{2K_{elastic}}$

$$\Delta_2 = 5.98 - \frac{245}{\left(2395^{kN}/11^{mm}\right)} = 5.98^{mm} - 1.13^{mm} = 4.85^{mm}$$



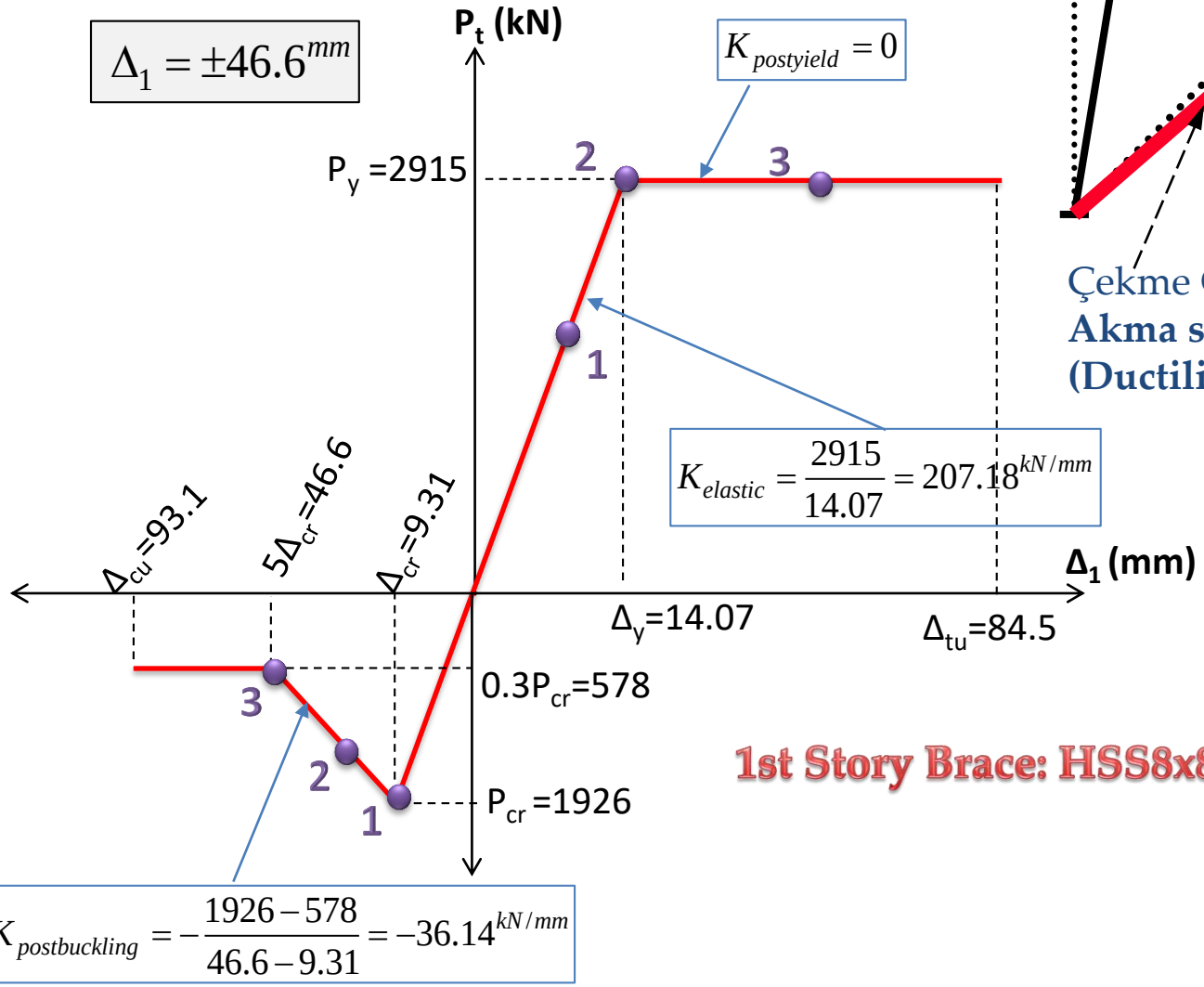


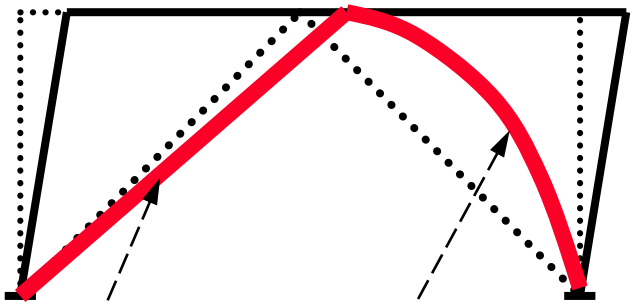
$$\Delta_{roof} = \Delta_1 + \Delta_2 = 46.6 + 4.85 = 51.45^{mm}$$

$$V_3 = V_2 + \Delta V = 2919 - 695 = 2224^{kN}$$

Statik İtme Analizi

3 Noktasında (Δ_1=46.6^{mm})





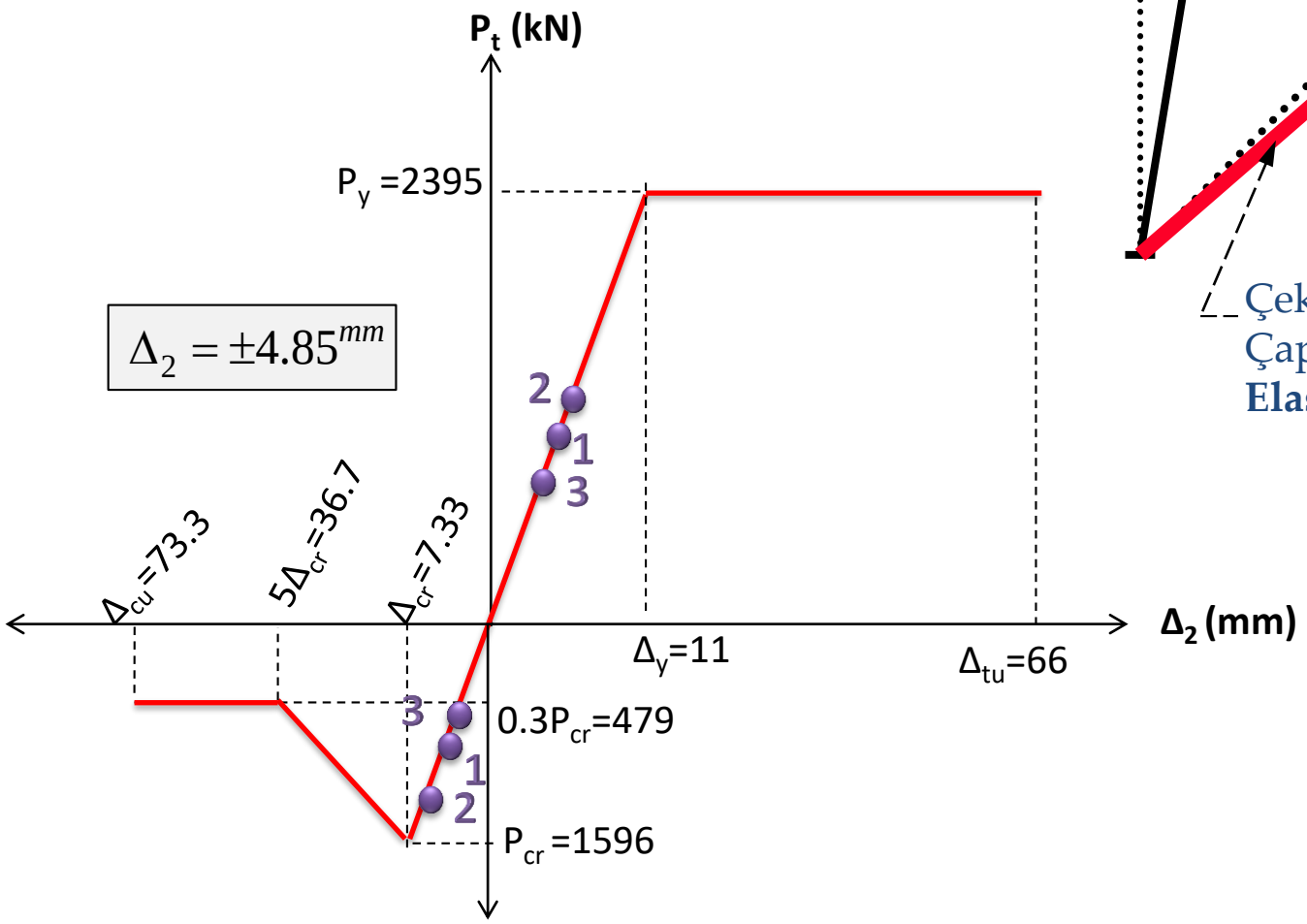
Çekme Çaprazı: **Akma sonrası def. (Ductility=3.3)**

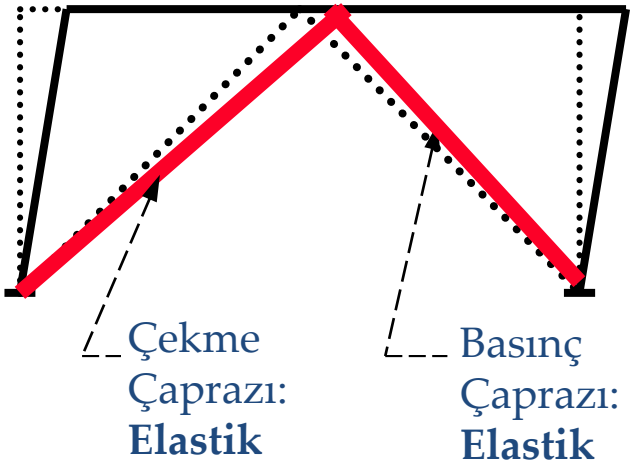
Basınç Çaprazı: **Ductility=5**

1st Story Brace: HSS8x8x5/8

Statik İtme Analizi

3 Noktasında (Δ_2=4.85^{mm})





2nd Story Brace: HSS8x8x1/2

Statik İtme Analizi

Step 4: Ultimate stage: 1. kat çaprazı $\Delta_{max}=\Delta_{tu}=6\Delta_y=84.5^{mm}$ ulaşır

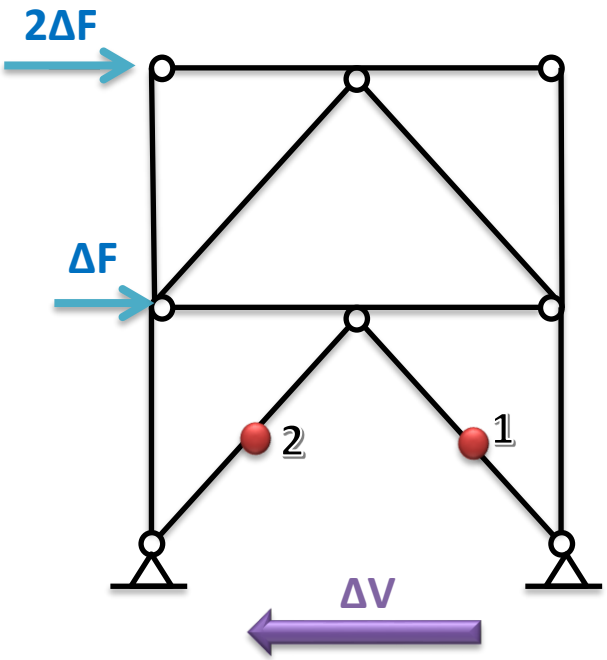
1. Katta $\Delta_{max}=84.5^{mm}$

Brace Forces: P_{R1} (Comp.) and P_{L1} (Tension)

When $\Delta_1=\Delta_{tu1}=84.5^{mm}$

$$P_{R1}\neq P_{L1}\Rightarrow \Delta P_{R1}+\Delta P_{L1}=\frac{3\Delta F}{\cos 51.3}$$

$$\left.\begin{array}{l} P_{L1}=P_y=2915^{kN}\rightarrow \Delta P_{L1}=0 \\ P_{R1}<P_{cr1}\rightarrow \Delta P_{R1}=0 \end{array}\right\} \Delta F=0 \qquad \Delta F=0$$



1. Kat Δ_{max} değerine ulaşır

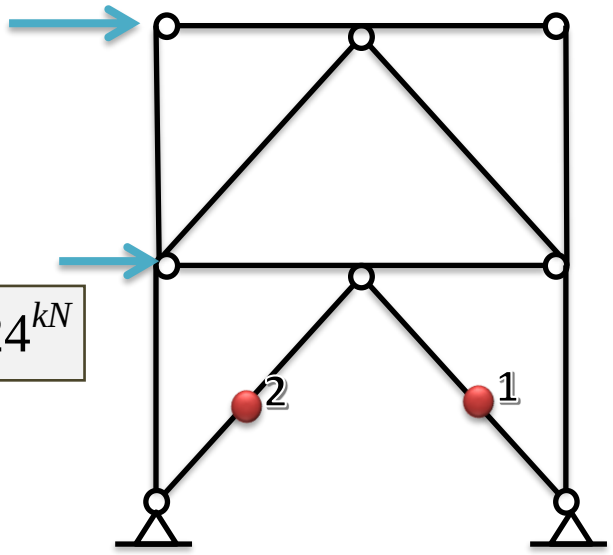
First Story Drift, $\Delta_1=\Delta_{1,max}=84.5^{mm}$

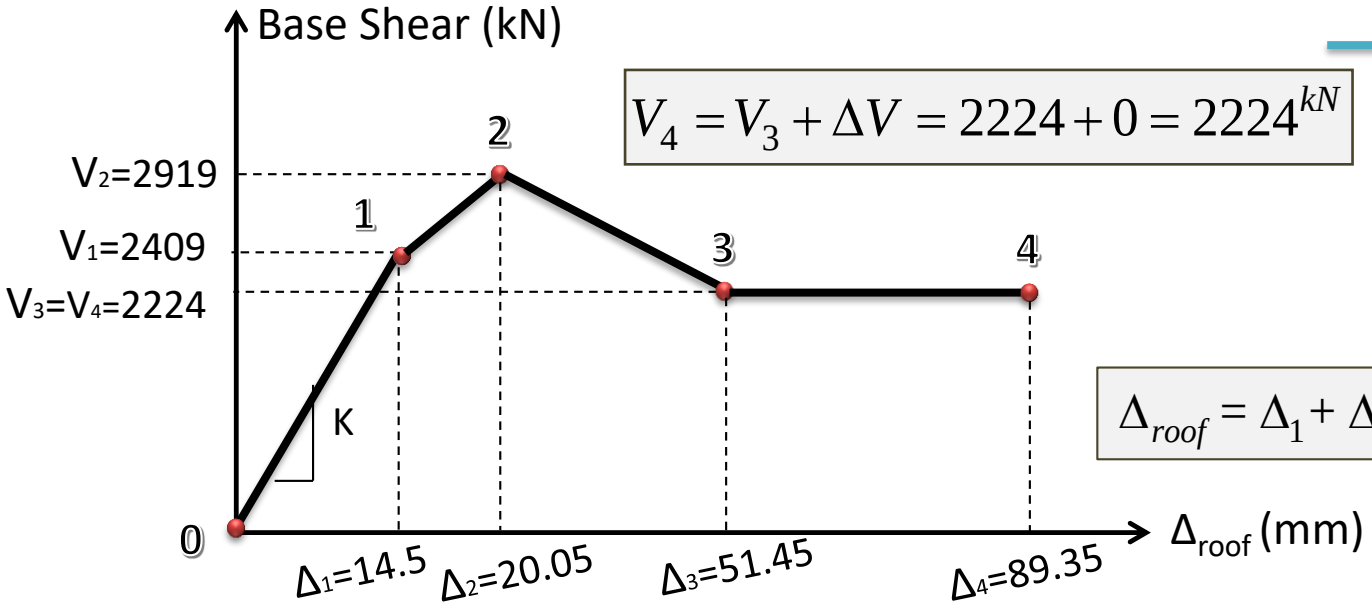
Second Story Drift, $\Delta_2=4.85+\frac{2\Delta F}{2K_{elastic}}$

$$\Delta_2=4.85+\frac{0}{\left(2395^{kN}/11^{mm}\right)}=4.85^{mm}$$

Statik İtme Analizi

Step 4: Son Adım (4 Noktası)





$$V_4 = V_3 + \Delta V = 2224 + 0 = 2224^{kN}$$

$$\Delta_{roof} = \Delta_1 + \Delta_2 = 84.5 + 4.85 = 89.35^{mm}$$

1. Kat Göreli Kat Ötelemesi Açısı

$$\text{Peak Story Drift Ratio } 1 = \frac{\Delta_{1,\max}}{h_1} = \frac{84.5}{5000} = 1.69\%$$

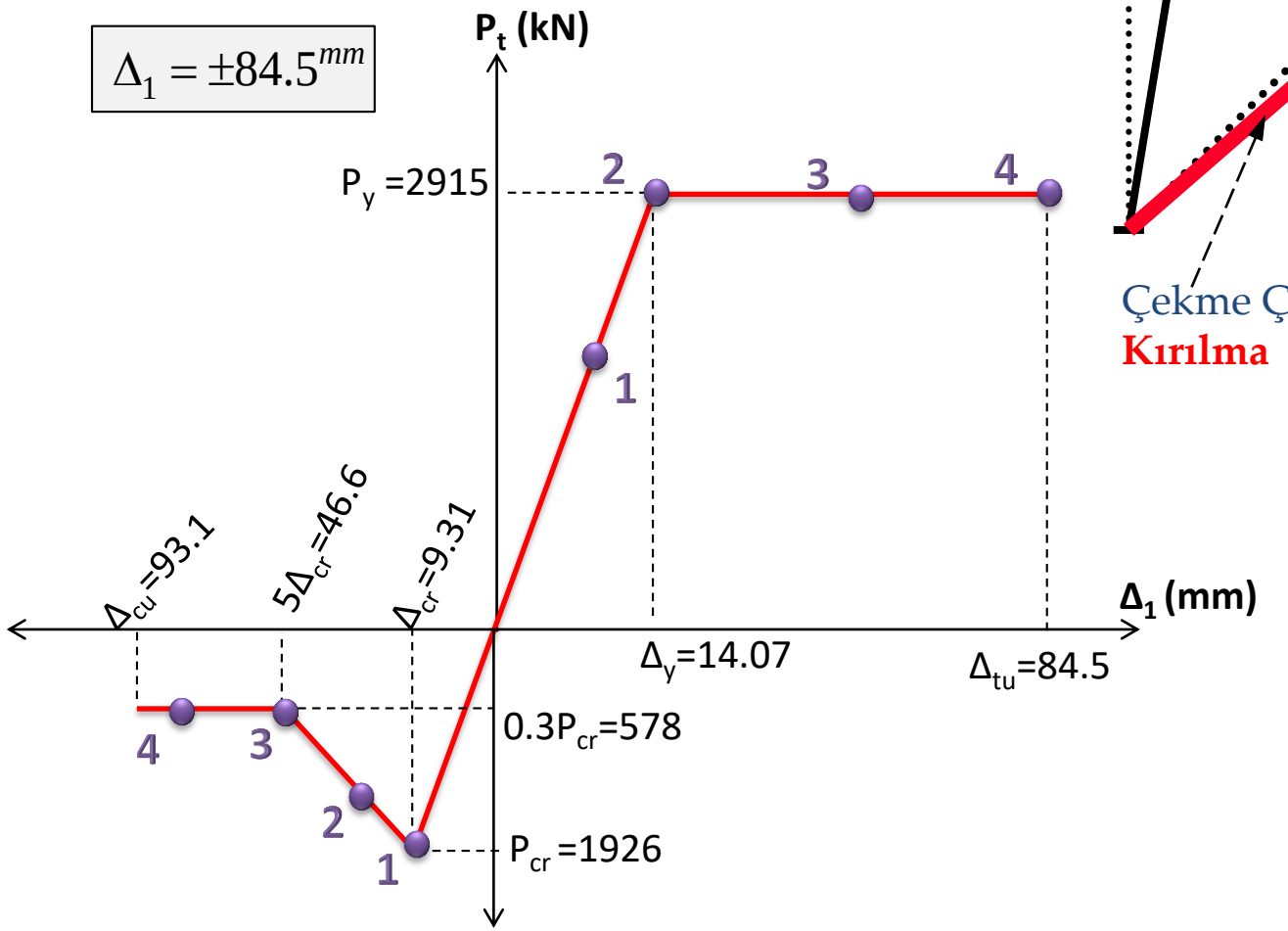
2. Kat Göreli Kat Ötelemesi Açısı

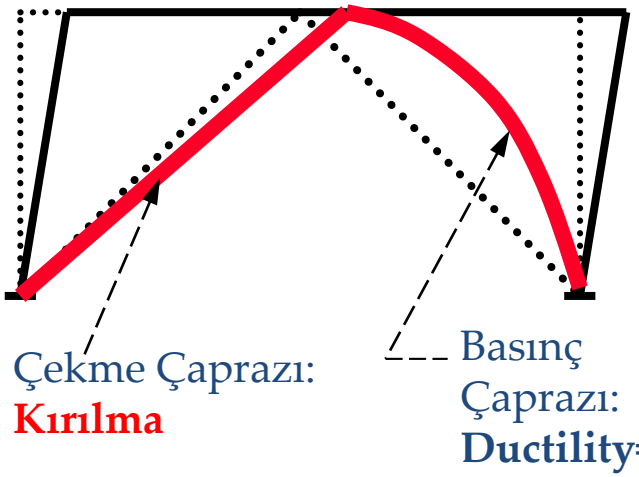
$$\text{Peak Story Drift Ratio } 2 = \frac{\Delta_{2,\max}}{h_2} = \frac{4.85}{4000} = 0.12\%$$

Statik İtme Analizi

4 Noktası (Δ_1=84.5^{mm})

$\Delta_1 = \pm 84.5^{mm}$



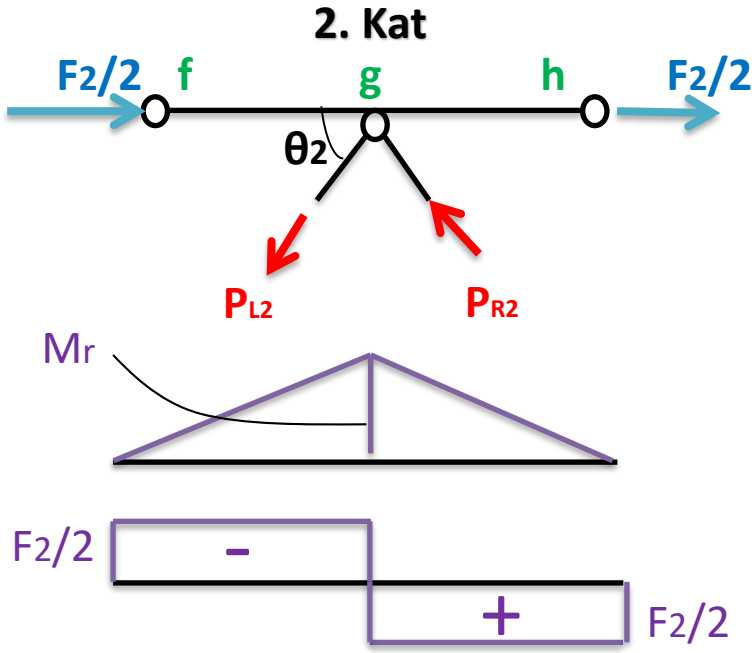


1st Story Brace: HSS8x8x5/8

Statik İtme Analizi

(5) Kirişler elastik kalıyor mu?

- İkinci kat çaprazları son aşamaya kadar elastik kaldığından, çapraz kuvvetleri eşit kaldı. Dolayısıyla, ikinci kat kirişindeki eğilme talebi sıfırdı.
- Maksimum eksenel kuvvet talebi, çapraz kuvvetleriyle doğrusal orantılıydı.



$$P_{L2} = P_{R2} = 1302^{kN} \quad \text{(Point 2)}$$

$$M_r = M_g = \frac{1}{4}(L)(P_{L2} - P_{R2})\sin\theta_2 = 0$$

$$F_2 = (P_{L2} + P_{R2})\cos\theta_2 = (1302 + 1302)\cos 45 = 1841^{kN}$$

$$P_r = F_2 / 2 = 1841/2 = 921^{kN}$$

Second Story Girder (W27x235)

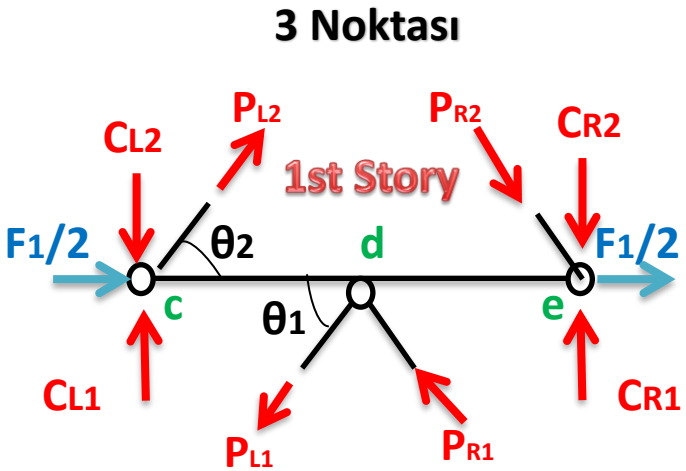
$$P_c = 14,314^{kN} > P_r = 921^{kN} \rightarrow OK$$

2. Kat Kirişi Elastik Kalıyor

Statik İtme Analizi

(5) Kirişler elastik kalıyor mu?

- Birinci kat kirişine gelen talepler, çapraz burkulması meydana geldikten sonra çapraz kuvvetlerine bağlı olarak değişmiştir. Bu nedenle, P-M etkileşimi açısından kritik noktalar öncelikle belirlenmelidir.
- En yüksek eğilme talebi, çapraz kuvvetleri arasındaki farkın maksimum olduğu noktada meydana gelmiştir. Bu nedenle, 3. (veya 4.) Noktadaki talep kritiktir.
- Kiriş üzerindeki en yüksek eksenel kuvvet talebine gelince, maksimum eksenel kuvvet 2. Noktada elde edilmiştir.



First Story Girder (W30x235)

$$\frac{P_r}{2P_c}+\frac{M_r}{M_c}\le 1.0 \rightarrow \frac{345}{2\times 14282}+\frac{3648}{4935}=0.75<1.0$$

$$\left.\begin{aligned} P_{L1}&=P_{y1}=2915^{kN}\\ P_{R1}&=0.3P_{cr1}=578^{kN}\\ P_{L2}&=P_{R2}=1056^{kN}\end{aligned}\right\}\ \text{(Point 3)}$$

$$M_r=M_g=\frac{1}{4}(L)(P_{L1}-P_{R1})\sin\theta_1$$

$$F_1=\left[\left(P_{L1}+P_{R1}\right)\cos\theta_1-\left(P_{L2}+P_{R2}\right)\cos\theta_2\right]$$

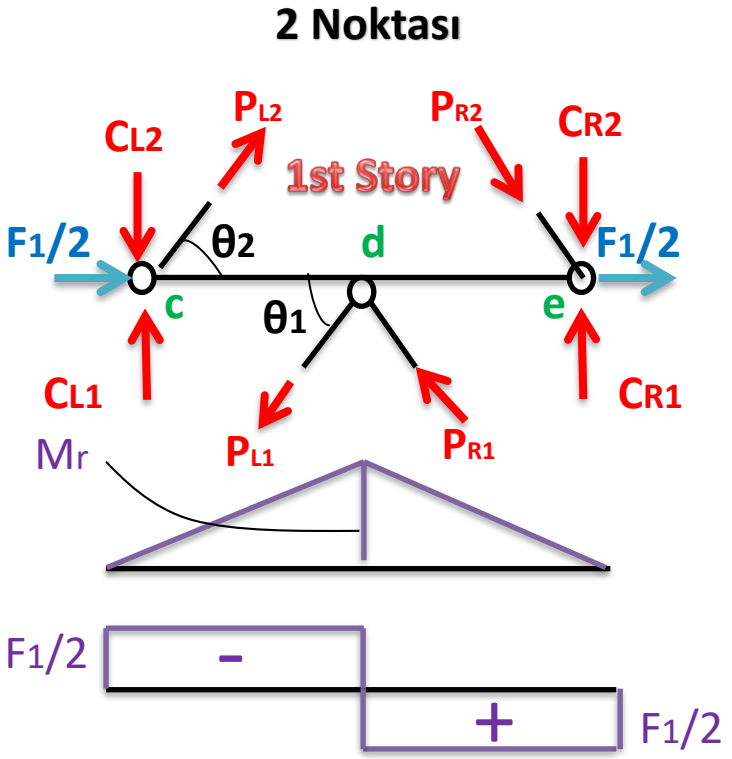
$$F_1=\left[\left(2915+578\right)\cos 51.3-\left(1056+1056\right)\cos 45\right]=691^{kN}$$

$$M_r=\frac{\left[(2915-578)\sin 51.3\right]\times 8}{4}=3648^{kNm}$$

$$P_r=F_2/2=691/2=345^{kN}$$

Two-Story One-Bay CBF

(5) Kirişler elastik kalıyor mu?



$$\left.\begin{aligned} P_{L1} &= P_{y1} = 2915^{kN} \\ P_{R1} &= 1754^{kN} \\ P_{L2} &= P_{R2} = 1302^{kN} \end{aligned}\right\} \text{(Point 2)}$$

$$M_r = M_g = \frac{1}{4}(L)(P_{L1} - P_{R1})\sin\theta_1$$

$$F_1 = \left[\left(P_{L1} + P_{R1}\right)\cos\theta_1 - \left(P_{L2} + P_{R2}\right)\cos\theta_2\right]$$

$$F_1 = \left[\left(2915 + 1754\right)\cos 51.3 - \left(1302 + 1302\right)\cos 45\right] = 1078^{kN}$$

$$M_r = \frac{\left[(2915 - 1754)\sin 51.3\right]\times 8}{4} = 1812^{kNm}$$

$$P_r = F_2 / 2 = 1078 / 2 = 539^{kN}$$

First Story Girder (W30x235)

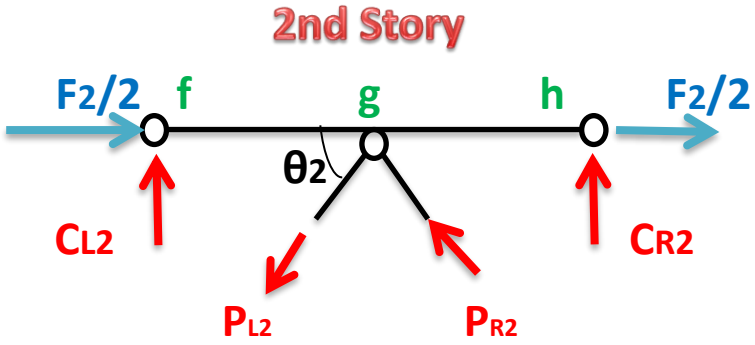
$$\frac{P_r}{P_c} = \frac{539}{14282} = 0.04 < 0.2$$

$$\frac{P_r}{2P_c} + \frac{M_r}{M_c} \le 1.0 \rightarrow \frac{539}{2\times 14282} + \frac{1812}{4935} = 0.34 < 1.0$$

1. Kat Kirişi Elastik Kalıyor!

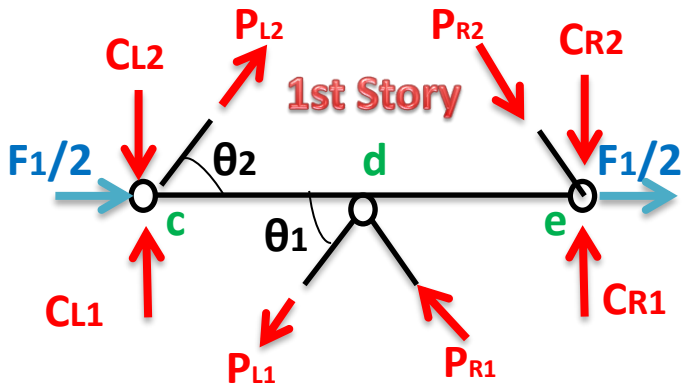
Two-Story One-Bay CBF

(6) Kolonlar elastik kalıyor mu?



$$\left.\begin{aligned}\sum M_g = 0 \rightarrow C_{L2} = C_{R2}\\ \sum F_y = 0 \rightarrow C_{L2} + C_{R2} = (P_{L2} - P_{R2})\sin\theta_2\end{aligned}\right\} C_{L2} = C_{R2} = 0$$

2. Kat Kolonu Elastik Kalıyor!



$$C_{L1} = C_{L2} - P_{L2}\sin\theta_2 + \frac{(P_{L1} - P_{R1})\sin\theta_1}{2}$$

$$C_{R1} = C_{R2} + P_{R2}\sin\theta_2 + \frac{(P_{L1} - P_{R1})\sin\theta_1}{2}$$

First Story Column (W14x132)

$P_c = 7544^{kN} > P_r = 1736^{kN} \rightarrow OK$

1. Kat Kolonu Elastik Kalıyor!

At Point 3

$$\left.\begin{aligned}P_{L1} = P_{y1} = 2915^{kN}\\ P_{R1} = 0.3P_{cr1} = 578^{kN}\\ P_{L2} = P_{R2} = 1056^{kN}\end{aligned}\right\} \text{(Point 3)} \Rightarrow \left\{\begin{aligned}C_{L1} = 0 - 1056\sin 45 + \frac{(2915-578)\sin 51.3}{2} = 165^{kN}\ (Comp.)\\ C_{R1} = 0 + 1056\sin 51.3 + \frac{(2915-578)\sin 51.3}{2} = 1736^{kN}\ (Comp.)\end{aligned}\right\}$$

Teşekkürler...

oseker@gtu.edu.tr

Referanslar

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